STATUS REPORT 3

PROJECT TITLE: LOCAL AND GLOBAL ACOUSTIC CONTROL FOR LAUNCH VEHICLE PAYLOAD FAIRINGS

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ACCOMPLISHMENTS SINCE PREVIOUS PROGRESS REPORT:

Developed a control model of a tapered waveguide for the purpose of understanding the controllability.

DISCUSSION

The results reported in progress report 2 motivated us to develop a control model of a tapered waveguide. The model is an idealization of the convergent end of the fairing. Experimental results obtained at Duke University demonstrated the ability of a single control speaker to produce global attenuation in the shroud. Our present objective is to develop a mathematical model of a tapered waveguide and determine the relationship between fairing geometry and modal controllability.

A schematic of the fairing geometry we are considering is shown in Figure 1. One possible method for approaching the wave propagation within such an enclosure is the generation of a piecewise model. Equating pressure and velocity at boundary conditions would be used to enforce some level of continuity between the sections. For this project we chose to analyze the first of the three sections that comprise the fairing cavity. If this model were to be used within the actual research, each of the three sections simultaneously to insure that the boundary conditions are met. However, for the purposes of this project, we chose to consider the case of a truncated cone having rigid end conditions at both the major and minor ends of the enclosure, as shown in Figure 2. For the purposes of studying controllability, we assumed an external disturbance was acting within the enclosure, with an input located at the minor end of the system to facilitate actuation of the controller.

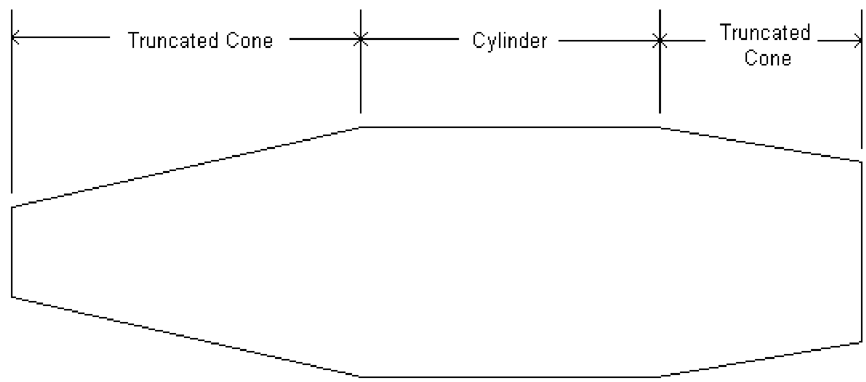

Figure 1. Representative geometry of typical payload fairings.

State-Space Modeling

To properly model the dynamics of this system, it is necessary to consider the generalized wave equation as follows:

$$\gamma P_o \frac{\partial}{\partial x}\left(A(x)\frac{\partial P(x,t)}{\partial x} \right) = \rho A(x)\frac{\partial^2 P(x,t)}{\partial t^2} \qquad (1)$$

where γ = ratio of specific heats, 1.40

Po = ambient pressure within the enclosure, 101.3 E 3 Pascals

ρ = density of air, 1.206 kg/m3

A(x) = cross-sectional area of cone

P(x,t) = Pressure as a function of location and time

For cylinders, the extreme case of a conic section with L = inifinity, the cross-sectional area is considered constant, allowing it to be extracted from the partial derivative with respect to x. This facilitates a simplified version of the wave equation. However for the case of a conical bore, A(x) is a function of location and therefore cannot be removed from the partial derivative. Given the geometry shown in figure 2, the actual equation for cross-sectional area becomes the following:

$$A(x) = A\left(1 - \frac{x}{L}\right) \tag{2}$$

Upon initial inspection, the wave equation takes a form similar to that of the longitudinal vibration in a tapered rod.

$$\frac{\partial}{\partial x}\left(EA(x)\frac{\partial \omega(x,t)}{\partial x}\right) = \rho A(x)\frac{\partial^2 \omega(x,t)}{\partial t^2} \tag{3}$$

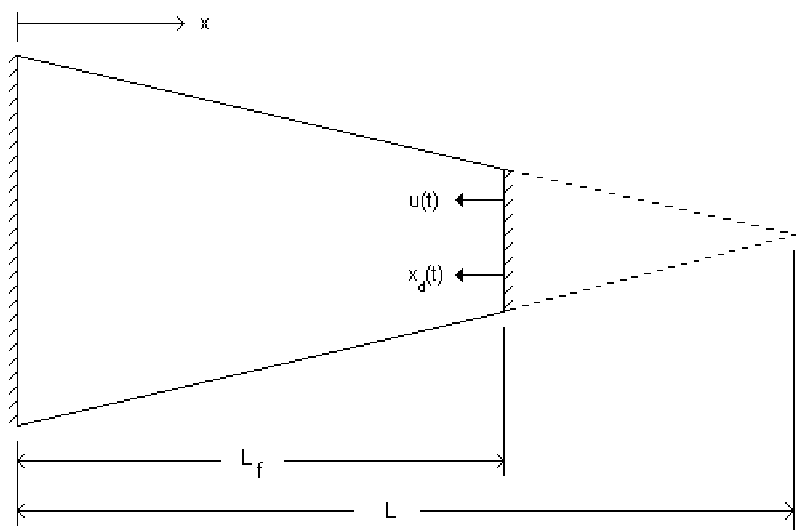

Figure 2. Truncated cone to be studied. (L = 1.75 m, L_f = 1.00 m)

Comparing terms between the governing equation (Eqn. 1) for spherical wave propagation, with the governing equation (Eqn. 3) for longintudinal vibration in tapered rods, it can be seen that each side of the equation is roughly equivalent to the corresponding term in the other equation. Equations 4 and 5 illustrate this comparison:

$$\frac{\partial}{\partial x}\left(EA(x)\frac{\partial \omega(x,t)}{\partial x}\right) \approx \gamma P_o \frac{\partial}{\partial x}\left(A(x)\frac{\partial P(x,t)}{\partial x}\right) \tag{4}$$

$$\rho A(x)\frac{\partial^2 \omega(x,t)}{\partial t^2} \approx \rho A(x)\frac{\partial^2 P(x,t)}{\partial t^2} \tag{5}$$

By equating the "kinetic" (Eqn 4) and "potential" (Eqn. 5) terms of each equation, it is possible to view the previously ambiguous components of the wave equation in a way that is comparable to to that of vibrations within a solid rod. Following the same method for solving the tapered bar problem, the pressure variable is reduced to two components through separation of variables, a spatial and a temporal solution.

$$P(x,t) = \phi(x)\eta(t) \tag{6}$$

From this point, a Rayleigh-Ritz approximation method is used to obtain a series representation of the spatial solution to the wave equation. Using the wave propagation solution for a closed-closed tube, the following admissible function was selected as a base point for the approximation method

$$\begin{gathered}\phi_i(x) = \cos\left(\frac{i\pi x}{L}\right) \\ i = 1,2,\ldots,n\end{gathered} \tag{7}$$

where $\phi_i(x)$ is the admissible function.

Following the derivation of Meirovitch [1997], equivalent stiffness and mass terms were developed in accordance with the following equations:

$$k_{ii} = 2\gamma P_0 A\left(\frac{i^2\pi^2}{L^2}\right)\int_0^{L_f}\left(1-\frac{x}{L}\right)\left[\partial_x \phi_i(x)\right]^2 dx \tag{8}$$

$$k_{ij} = 2\gamma P_0 A\left(\frac{i\pi}{L}\right)\left(\frac{j\pi}{L}\right)\int_0^{L_f}\left(1-\frac{x}{L}\right)\left[\partial_x \phi_i(x)\right]\left[\partial_x \phi_j(x)\right] dx \tag{9}$$

$$m_{ii} = 2\rho A\int_0^{L_f}\left(1-\frac{x}{L}\right)\left[\phi_i(x)\right]^2 dx \tag{10}$$

$$m_{ij} = 2\rho A\int_0^{L_f}\left(1-\frac{x}{L}\right)\left[\phi_i(x)\right]\left[\phi_j(x)\right] dx \tag{11}$$

Applying the corresponding indices, i=1,2,3 and j=1,2,3, generalized mass and stiffness matrices were obtained, each of these associated with the first three acoustic modes of the truncated conical bore shown in figure 2. Solving for the eigenvalues and eigenfunctions associated with these equations, the following expression was obtained for the pressure variation within the cavity.

$$
\begin{aligned}
P(x,t) = &\left[-0.05192\cos\left(\frac{\pi x}{L}\right)+0.26466\cos\left(\frac{2\pi x}{L}\right)-0.96294\cos\left(\frac{3\pi x}{L}\right)\right]\eta_1(t)+ \\
&\left[-0.03926\cos\left(\frac{\pi x}{L}\right)+0.96296\cos\left(\frac{2\pi x}{L}\right)+0.26678\cos\left(\frac{3\pi x}{L}\right)\right]\eta_2(t)+ \\
&\left[-0.99788\cos\left(\frac{\pi x}{L}\right)-0.051662\cos\left(\frac{2\pi x}{L}\right)+0.03960\cos\left(\frac{3\pi x}{L}\right)\right]\eta_3(t)
\end{aligned}
\qquad (12)
$$

Again, it must be said that this equation is composed of only the first three modes of vibration. A more complete solution could be obtained by increasing the number of modes included, accomplished through the generation and analysis of larger mass and stiffness matrices.

While the pressure equation is an important component in understanding the behavior of acoustics within the enclosure, for controllability purposes, it is more appropriate to represent the system in the following form.

$$M\ddot{\eta}(t)+K\eta(t)=U \qquad (13)$$

Where M and K are the equivalent mass and stiffness matrices, and U is a vector of inputs used to incorporate actuator forces. Rearranging the coefficients as follows:

$$\ddot{\eta}(t)=-M^{-1/2}KM^{-1/2}\eta(t)+M^{-1/2}UM^{-1/2} \qquad (14)$$

A state space model of the system can easily be obtained.

Selecting six states to represent the system, three associated with the temporal component of each mode, $\eta_1(t)$, $\eta_2(t)$, $\eta_3(t)$, and three more associated with the time derivative of these first three states.

$$\begin{aligned} x1 &= \eta_1(t) \qquad x3 = \eta_2(t) \qquad x5 = \eta_3(t) \\ x2 &= \dot{\eta}_1(t) \qquad x4 = \dot{\eta}_2(t) \qquad x6 = \dot{\eta}_3(t) \end{aligned} \tag{15}$$

Generating the corresponding state matrices, the state-space representation is composed of the following set of matrices:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -6.12E5 & 0 & 2.85E6 & 0 & 7.99E6 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 2.85E6 & 0 & -2.65E7 & 0 & 3.75E7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 7.99E7 & 0 & 3.75E7 & 0 & 1.52E8 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 2.585 \\ 0 \\ -0.765 \\ 0 \\ 2.981 \end{bmatrix} \tag{16}$$

$$C = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 1 \end{bmatrix} \qquad D = [0]$$

An interesting characteristic of this system is the extreme range of values associate with the A matrix. Calculating the Q matrix associated with this system yields components ranging from 1 E0 to values on the order of 1 E16. Interestingly, this poses a problem when calculating the rank of the Q matrix. MATLAB utilizes 15-point precision in its calculations, such that the value of the smallest terms approaches 0, resulting in a rank less than the number of states in the system. To check the true rank of the system, it is possible to divide the first, third and fifth columns by a constant, reducing their order. If this constant is the same for all three columns, the system is simply scaled, and the rank can be solved for without errors introduced by numeric precision.

UPCOMING TASKS:

1. Explore observer-based control design for the fairing model to determine the potential sound reductions achievable with feedback control.