FRAMEWORKS THAT SUPPORT RESEARCH ON MATHEMATICS EDUCATION WITH COMPUTER ALGEBRA SYSTEMS: REFLECTIONS ON CAME SYMPOSIA

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Since the Hawaiian conference in 1995 (Berry et al, 1997) Computer Algebra in Mathematics Education *(CAME) has been established. Further to this (but not because of this!!) some of us have 'lost our innocence' along the way; youthful dreams of enhanced learning have been replaced by realism that all tools have their constraints as well as their enablements. CAME Symposia have been forums for these discussions. The rationale and goals for CAME Symposia are to serve as a bridge between two communities, the CAS research community and the main mathematics education community; to facilitate the dissemination and exchange of information on research and development in the use of computer algebra in mathematics education; to facilitate access to international expertise in the use of computer algebra in mathematics education; to promote the study of the use of computer algebra in mathematics education. The rationale for this discussion group is to present the products of CAME Symposia and to discuss their potential contribution to the PME community. In this paper we first describe briefly issues that were dealt with in each symposium. We then outline the specific topics for discussion.*

The First CAME Symposium

The first CAME Symposium was held at the Weizmann Institute of Science, Rehovot, Israel in August 1999 (see link to Weizmann, 1999) following on from PME 23. The theme was: *Exploring CAS as a pedagogical vehicle towards expressiveness and explicitness in mathematics*. Plenary speakers were: Jean-baptiste Lagrange, Steve Lerman , Edith Schneider, Ted Eisenberg, Paul Drijvers, Kaye Stacey, Nurit Zehavi and Anna Sierpinska, John Berry, Richard Noss and Amitai Regev. We wanted to address two issues in particular:

1. the links (or lack of) between "theoretical" work in mathematics education and classroom practice, with particular respect to the role of CAS;
2. the place of CAS research and CAS-related activities within mathematics education research as a whole.

It is a long-standing problem in mathematics education to connect research and classroom practice; as CAS technology increasingly impacts on mathematics curricula, the challenges and opportunities for mathematics education to inform curriculum change are considerable. The CAS-related research questions that we wished to highlight at the workshop had to do with what we labeled explicitness and expressiveness. In using a CAS, a particular explicit symbolism is forced: each input requires a particular forced way of viewing things and expressing relationships and the output needs to be interpreted similarly. CAS has the potential to provide expressive powers for its users; it is possible to express ideas (mental objects) in a concrete form (visible objects). It seems reasonable to assume that in the tensions set up between thoughts and explicit expressions there is considerable scope for researchers to understand mathematical learning better. At the conference presenters from four countries shared their experience and research direction in using CAS for teaching. Colleagues from the PME community reacted to the papers and together with the participants the ground was set for further work.

Lloyd, G. M., Wilson, M., Wilkins, J. L. M., & Behm, S. L. (Eds.). (2005). *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.*

In this first Symposium Jean-Baptiste Lagrange introduced many people to a new approach to CAS work. This approach used the anthropological framework of Chevallard and an "instrumentation" approach to tool use. This theme has been a constant feature of all subsequent CAME Symposia.

The Second CAME Symposium

The second CAME Symposium was held in July 2001 at the Freudenthal Institute, Utrecht, The Netherlands with the theme: *Communicating Mathematics through Computer Algebra Systems* (see link to Freundenthal, 2001). The symposium examined research on the relation between techniques and conceptual understanding, on the role of the teacher, and on the affordances of technology in realizing specific pedagogical approaches. Plenary speakers were: Michèle Artigue, Neil Challis, Koeno Gravemeijer, Kathleen Heid, Kenneth Ruthven, Kaye Stacey, Michal Yerushalmy and Rose Mary Zbiek. As a natural continuation of the previous symposium we began with the issue of the subtlety of the relationship between paper-and-pencil techniques, CAS techniques and conceptual understanding. The nature of paper-and-pencil techniques is different from that of CAS techniques. How do these different kinds of techniques interact with concept development and understanding? What is the nature of the "instrumentation" process, during which a tool gradually develops into an instrument, for learning to do mathematics using a computer algebra system? Another perspective was presented by Kathleen Heid who shared with the participants her view of how theories about the learning and knowing of Mathematics can inform the use of CAS in school mathematics.

Friendly personal relationships contrasted strongly with deep theoretical divisions between many of the participants (you don't come to CAME for an easy intellectual ride). Even with a topic as focused as 'CAS and teachers' (presented by Stacey and Zbiek) it was clear that the different interests of the group members created different foci and questions. For example, members who were involved in in-service work with teachers wanted to know how best to train teachers whilst curriculum developers focused on how teachers might help students. At the core, however, everyone was united in trying to understand the phenomena of teachers using CAS.

The Third CAME Symposium

The third symposium was held at the IUFM in June 2003 in Reims, France with the theme: *Learning in a CAS Environment: Mind-Machine Interaction, Curriculum & Assessment (*see link to Reims, 2003). Plenary speakers were: Lynda Ball, Roger Brown, Al Cuoco, Peter Flynn, Celia Hoyles, Colette Laborde, David Leigh-Lancaster and Luc Trouche. The themes were a little bit more discrete than they were in CAME 2 reflecting, perhaps, work that different groups were immersed in.

Most of the members of the working group on 'Assessment' were also involved in CAS-related curriculum development that involves assessment with CAS and so a good deal of discussion was on very practical matters. Five members of the group were from Victoria, Australia, where the *Computer Algebra in Schools: Curriculum, Assessment & Teaching Project* has been ongoing. This project has been followed with interest by a number of people and there were many questions about this in the working group. For example: How CAS in assessment forces us to examine the goals of teaching and learning mathematics?

Strangely enough the mathematics curriculum itself emerged as a point of heated debate (strange that this division had not occurred earlier). Some argued that there is a strong need for a general outline of a curriculum where the enhancements in technology, especially the presence of computer algebra systems and dynamical geometry environments, will be incorporated. Others

suggested that we need a curriculum where the choice of technology used is not the central factor.

This ends our brief description of the first three CAME Symposia. We now explore various themes in greater depth. These are: CAS-based curricular materials; CAS and teachers; CAS and assessment; CAS, the instrumental approach and orchestration. We end by outlining our intentions towards the Fourth CAME Symposium.

CAS-based Curricular Materials

The availability of CAS for microcomputers in the mid-eighties led some teachers and researchers to explore new teaching methods that utilize CAS to enhance the teaching and learning of mathematical topics. One notable strategy was to allow students to concentrate on conceptual aspects as they learn a topic. Early CAS-based studies by Heid (1988) and others indicated that re-sequencing of the content, so that the concepts are taught before the manipulation skills, was effective for achieving a greater understanding of concepts without decreasing the achievement of manipulation skills. As more mathematics teachers became familiar with CAS, a new area surfaced, namely the design of CAS-based written support materials. Researchers and curriculum designers realized, through experimental projects, that learning to utilize CAS to construct mathematical meaning is both complex and insightful. As more studies were carried out, more questions were asked: What tasks involving CAS engage students in conceptual aspects? How do we assess their quality? What is the role of CAS in emerging curricula? Such questions were addressed in the first CAME Symposium. Didactical works with CAS in France, Austria, The Netherlands, Australia and Israel were reported and analyzed.

Jean-baptiste reported on his involvement in an experimental project of the French ministry of Education for the teaching of pre-calculus with CAS. He argued that focusing on an opposition between skills and concepts does not help to understand changes introduced by CAS use in teaching and learning. He proposed to think rather of the contribution of techniques to students' understanding and to consider the impact of CAS at this level: obsolescence of paper/pencil techniques, potential contribution of new CAS techniques. He presented the notion of "praxeology" as means to consider tasks, techniques and theorizations in students' processes of conceptualization. This point was further elaborated by Steve Lerman who reacted to the French work. Lagrange's paper has been a prelude to ongoing analysis and discussion of the role of techniques in CAS work.

Edith Schneider and her Austrian colleagues were trying to get mathematics teachers interested in using CAS in their classrooms by working with the individual teacher to develop materials specifically for them to use. She reported an effective change of atmosphere of learning in the experimental classes, where the role of the teacher was less as the source of knowledge and more as a collaborator in helping students discover knowledge for themselves. However, the researchers realized that the teachers were unable to take a deeper, more critical examination of the fundamental didactic questions on the potential of CAS in teaching. Ted Eisenberg, in his reaction paper, emphasized that teachers are not curriculum developers, thus they should get mock lessons from experts. He expressed his skepticism strongly by stating that the Austrian team was years ahead of themselves.

Paul Drijvers from the Freundenthal Institute described the role of the theoretical framework of Realistic Mathematics Education and developmental research in developing and performing

an experiment using symbolic calculators in senior secondary mathematics classes. He provided rich data that helped him address the following questions:

1. Is it possible to re-sequence a course using CAS, so that concept development precedes the solving techniques and algorithms?
2. Can algebraic insight improve because of CAS use?
3. What obstacles do students experience while working with computer algebra?

Kaye Stacey discussed these questions, drawing on experiences from experiments that she and her students carried out at the University of Melbourne. We will say more about Australian work in the next section.

Nurit Zehavi and Giora Mann from Israel presented a study that attempted to investigate (a) the role of CAS in modeling word problems, and (b) the role of algebraic expressions involving parameters in making explicit the underlying structure and constraints of a family of story problems. The research was done within the formative development stage of the MathComp project that was initiated in 1996, and aimed to integrate CAS into teaching in junior and senior high school. The learning process was organized through a careful design of problems and tasks for the students as a group and planning of the activities with the teachers. The students' tasks were not only to solve problems given by the teacher but also to invent their own problems. Anna Sierpinska, in her reaction paper, proposed that it was this feature of the didactic situation, and not so much the availability of a CAS, that was responsible for the students' progress in their thinking about modeling word problems using equations and conditions on variables. At the end of the symposium it was clear that the mutual impact of CAS use and task design would be further explored in the following symposia.

The group discussion that followed the plenary lectures by Michèle Artigue and Ken Ruthven in the 2nd symposium, dealt with the difficulties in developing criteria for CAS-based tasks, and proposed several suggestions to the question: What are the implications of the complexity of instrumentation for task design and research in CAS environments? Here are some of the participants' suggestions for designing tasks and the associated research.

TASKS	*RESEARCH*
Tasks designed to address known difficulties of students	Are the difficulties encountered in a paper and pencil environment actually present in a CAS environment?
Diverse tasks designed for conceptual development	Examine the relationships between techniques and concepts when using CAS and other technologies. Encourage the implementation of tasks under different conditions (to obtain an international perspective).
Tasks designed to integrate the different types of representations available in a CAS environment	What role does visualization play in different environments? Under what circumstances is it appropriate to work with two or more representations?

Tasks that encourage a good classroom discussion and for which the resulting discussion opens 'webs of meaning' (Noss & Hoyles (1996)) that go beyond that which occurs within the regular curriculum.	Exploring the opportunities to extend the curriculum. Carry out classroom experiments with rich tasks that would not really be feasible without the CAS.
Tasks that motivate socio-mathematical norms that arise in the new mathematical environment	Examine the types of communication and accounting in a CAS classroom. Examine the tension between the cognitive and cultural aspects.
Tasks for assessment of the impact of integrating CAS into teaching	Identify those difficulties created by the complexity of instrumentation when students are using the CAS. Compare students' achievement when implementing alternative approaches to the use of CAS.
Tasks specially designed for teachers to experience doing mathematics with CAS	Examine the process of teachers' development in introducing CAS. Examine the transition of teachers from graphic calculators to the CAS culture. Classify and analyse tasks developed by teachers who are experienced users of CAS.

Consequently, at the 3rd symposium one of the themes dealt with Curriculum and Task Design. In the plenary session for this theme, Al Cuoco presented his and Paul Goldenberg's paper, *CAS and Curriculum: Real Improvement or Deja Vu All Over Again,* and Collete Laborde presented a reaction paper *The design of curriculum with technology: lessons from projects based on dynamic geometry environments*. Both enriching talks presented a solid framework for a very lively and practical discussion.

Perhaps the most important observation was that nowadays we see just the fragments of a new technology-based curriculum that is emerging. The group agreed that technology makes changes in the curriculum necessary. "Are we better with gradual changes of the existing curricula or should we try to make a new curriculum from scratch?" Of course a fully "revolutionary" approach is not possible due to social, economic, political and other factors, but nevertheless it could be interesting and beneficial if we could see what such a curriculum looks like. This discussion will be taken one step further by the topic group on "The impact of CAS on our understanding of mathematics education", at the 4th CAME Symposium. The theme will deal with questions such as: is the cognitive availability of operative knowledge and skills essential for mathematics education in spite of CAS? have the meanings of operative knowledge and skills changed so that these aspects of mathematics now require an ability to work with CAS? Getting a better understanding of these issues may enable the CAS-in-education community to better communicate with teachers, who are often reluctant to integrate CAS in teaching in spite of its availability and student familiarity with technology.

CAS and Teachers

CAS and Teachers was a theme of the 2001 Symposium. Teachers, of course, have a crucial mediational role in students' learning (with or without CAS). However, and as the meta study of Lagrange et al. (2001) note, CAS studies in the period 1994-1999 generally do not discuss the

role of the teacher. For our Symposium we wanted speakers who could broaden our understanding of teachers and CAS. We were aware of interesting work going on in Australia and in the USA and this is where we turned.

Both Australia and the USA have federal governments[1]. Within Australia, Victoria was the centre of a synergy between local government CAS initiatives (see the section on Assessment for more on this) and academic study around these initiatives. At the University of Melbourne Kaye Stacey was supervising the PhD theses of two experienced teachers, Margaret Kendal and Robyn Pierce. Kaye's presentation acknowledged the contributions of Margaret and Robyn. In the USA Kathy Heid's work stands out for its longevity and productiveness. Kathy is clearly someone who likes to work in a team and an important co-worker was Rose Mary Zbiek. We were aware of Kathy and Rose Mary's work on CAS and teachers from their PME-NA papers (Heid, 1995; Zbiek, 1995) and thought that Rose Mary would be an excellent reactor to Kaye's presentation.

Kaye's presentation focused on case studies of two teachers who had "adopted CAS as an extra technique for solving standard problems". Her assumption was that integrating CAS into teaching changes many aspects classroom practice "which teachers will make from the base of their prior teaching styles and their beliefs about mathematics and how it should be taught". One teacher focused on student understanding and restricted students' use of CAS. The other focused on CAS as an additional (to standard paper and pencil) technique for solving standard problems and emphasized time-saving routines. Issues raised included: different forms of classroom organization; the variety of approaches available to teachers; the increased range of methods available for solving problems and the tensions this raises for teachers; the different ways that teachers use graphic and symbolic calculators; implications for changes to the curriculum and assessment.

Rose Mary's reaction noted that "the route a teacher's journey takes depends on the individual's answers to a few non-trivial questions:

1. What is school mathematics?
2. What is mathematical understanding and how does it develop?
3. What is the teacher influence?"

Rose Mary introduced two other pairs of teachers, from the USA and from the UK, and noted situational constraints and enablements. However:

> Common to all scenarios is a professional development and research design that included observation data and artifacts as well as data about the teacher's belief, conception, and understanding of CAS, mathematics, and teaching.

She concludes that the critical essence of teaching with CAS in a time of transition is not CAS itself but "the extent to which it challenges values, abilities, practice and assumptions".

The day of discussion on the themes raised in the presentation and reaction noted that introducing CAS into teaching involved many groups of people, other than teachers, with distinct interests: students, parents, teacher trainers and curriculum developers. Further to this, but still regarding distinct interests, it was notable that individuals within the discussion group had different agendas with regard to teachers and CAS, e.g. members who were involved in in-service work with teachers wanted to know how best to train teachers whilst curriculum developers focused on how teachers might help students. These varied "interested parties" and the varied agendas strongly suggest that future considerations of teachers and CAS should be

[1] Do federal systems present fewer constraints on educational innovations such as the introduction of CAS?

placed within a wider "institutional" analysis. This links to a theme raised by Michèle Artigue at the Symposium "the life of a mathematical object in an institution" and the group recognised that it was important to understand teachers' "positioning" within their institutions.

CAS and Assessment

CAS and assessment was a theme of the 2003 Symposium but "assessment" is really too wide a term to describe the presentations and the debate. Important current themes in assessment such as formative assessment, dynamic assessment, adaptive assessment and e-assessment were not addressed. Further to this assessment of school and university level mathematics generally differ: in the classroom where schools generally have much smaller student numbers, and in final examinations where universities generally have greater freedom, including teachers setting the questions. The focus of the 2003 Symposium was really "CAS and senior high school high stakes examinations". We deviated from the CAME norm of presenter-reactor and had four presentations; all from Australians and three of these concerned with work going on in Victoria, Australia[2]. We briefly describe the four papers and then the group discussion.

Roger Brown presented a paper "Comparing system wide approaches to the introduction of Computer Algebra Systems into examinations" which looked at CAS-assumed examinations in Denmark and Australia. He addressed two questions:

1. How are the examination writers responding to the introduction of CAS?
2. What is the role of the CAS within the examinations?

He concluded that the use of CAS, in these case studies, results in surprisingly little difference in the types of questions used but that CAS greatly enhances the range of solution strategies available to students.

David Leigh-Lancaster reported on a CAS-assumed pilot study from the viewpoint of the Victorian Curriculum and Assessment Authority. This paper covered the design and development of the CAS pilot study and details of student performance in examinations. With regard to performance the CAS-cohort generally did as well or better than the non-CAS cohort. Pilot teachers and students, moreover, generally affirmed benefits of using CAS including greater depth treatment of existing material, access to new and interesting content and enhanced engagement, persistence and confidence of students.

Lynda Ball explored "Communication of mathematical thinking in examinations: A comparison of CAS and non-CAS student written responses". She examined students' written records in examinations including a comparison of written solutions for examination items common to CAS and non-CAS students. She found that CAS students generally gave shorter written solutions and noted a CAS-student tendency towards using a mixture of mathematical notation and words in documenting how problems were solved.

Peter Flynn presented a paper "Using Assessment Principles to Evaluate CAS-Permitted Examinations". The "principles" in question are three that pre-date the Victorian CAS pilot and relate to content, learning and equity in high-stakes assessment. They are, respectively, that assessment should: reflect the mathematics that is most important to learn; enhance mathematics learning and support good instructional practice; support every student's opportunity to learn important mathematics. With regard to CAS-assumed examinations Peter addressed a number of matters of concern. With regard to equity he explored different CAS calculators where there was

[2] http://extranet.edfac.unimelb.edu.au/DSME/CAS-CAT/ see: http://www.vcaa.vic.edu.au/vce/studies/mathematics/caspilot/casbackground.html

a differential effect on some items but, that over all questions, no one brand was of significant advantage. Matters raised in the day of discussion included:

- Whether examinations should have 'CAS free' sections or not.
- The difficulty of separating curriculum and assessment.
- The need to examine the goals of teaching and learning mathematics
- The need for further work on teachers' professional development
- The need to consider all types of assessment and not just timed examinations
- The need to be explicit about why particular questions are being set
- The problem that the aspect of CAS use envisaged in examination questions may be quite different to the aspects of CAS use that a teacher (and his/her students) focuses on.

CAS, the Instrumental Approach and Orchestration

CAS is a tool, a very complex tool, which incorporates various computational media (Cuoco, 2002). In their quest to understand CAS-as-a-tool CAS researchers turned to French research on *instrumentation.* V_rillon & Rabardel (1995) distinguish between a tool, as a material object, and an instrument as a psychological construct: "the instrument does not exist in itself, it becomes an instrument when the subject has been able to appropriate it for himself and has integrated it with his activity".

At the third symposium, Luc Trouche provided a clear exposition of the psychological foundations of instrumentation. He offered to consider an instrument as an extension of the body "made up of a tool component (a tool, or a fraction of a tool mobilized in the activity) and a psychological component." He emphasized the subject-tool dialectic and names the direction of the influences: *instrumentation*, how the tool shapes the actions of the tool using subject; *instrumentalisation*, the ways the subject uses (shapes) the tool. The psychological component was explained via the Piagetian notion of a *scheme* "the structure or organization of actions as they are transferred or generalized by repetition in similar or analogous circumstances" (Piaget & Inhelder, 1969, p.4). The evolution of this dialectic between tool and scheme is called "instrumental genesis". This is a complex process over time which links the enablements and constraints of the tool to the agent's prior understandings and activity. Trouche introduces the notion of "gestures". Gestures are observable behaviours and a "scheme is the psychological locus of the dialectic relationship between gestures and operative invariants, i.e. between activity and thought."

Trouche also introduced the term *instrumental orchestration* "to point out the necessity (for a given institution – a teacher in his/her class, for example) of *external steering* of students' instrumental genesis." He started from a study of a particular technological classroom environment as one of many possible forms of orchestration. This environment included students with TI-92s and exercise books, a rotating (amongst the class) "sherpa" student who operates the viewscreen, a viewscreen, a blackboard, specific tasks and a teacher. He noted the instrumental genesis of students via change in their mathematical behavior and their tool trajectories throughout the instrumental process. In commenting on how to support instrumental genesis he argued for strong teacher involvement in the instrumental process and recognition of the constraints and potential of the artifacts and of student behavior.

Hoyles broadened Trouche's discussion of orchestration to elaborate the role of artifacts in the process, describing how the notion of situated abstraction could be used to make sense of the evolving mathematical knowledge of a community as well as an individual. She concluded by elaborating the ways in which technological artifacts can provide shared means of mathematical

expression, and discussed the need to recognize the diversity of student's emergent meanings for mathematics, and the legitimacy of mathematical expression that may initially diverge from that of institutionalised mathematics.

Towards the Fourth CAME Symposium in Tandem with PME-NA

> Computer algebra use in mathematics teaching and learning is in its infancy. Nevertheless there are many teachers and educationalists who have integrated CAS into their teaching or conducted research into student understanding with CAS or who have led curriculum/assessment projects involving CAS use. (*Introduction,* Berry et al., 1997)

These words were written after the 1995 ACDCA symposium in Honolulu. Ten years later, innovative classroom uses of CAS still exists, but progress has not been as rapid as many expected. Successes, and also difficulties, have provided opportunities for conceptualizing the complex impact of technology on classroom processes. CAS is not the most popular technology in classrooms. It is challenged or complemented, even in algebra, by numerical or geometrical environments (spreadsheets, dynamic geometry) and by web based applications. As one technology amongst others, CAS appears to have very rich and complex links to mathematical understanding and practices. It is therefore not surprising that didactical conceptualizations, outlined above, that were necessary in order to analyze these links are now recognized as very useful even with regard to other technologies.

In the 4th CAME Symposium we plan to further examine these conceptualizations, beginning with the issues of instrumentation and praxeologies. We will also deal with the mathematics curriculum, namely, the impact of CAS on our understanding of mathematics education, and again try to examine what teachers learn while teaching with CAS. In the PME-NA discussion group, through this paper and subsequent discussions, we hope to familiarize participants with changes in the goals, content, methods and forms of social interaction in mathematics teaching that computer algebra affords and, at the same time, help the CAME community in its efforts in shaping research and development of computer algebra in mathematics education.

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