VIDEO-BASED RESEARCH ON MATHEMATICS TEACHING AND LEARNING: RESEARCH IN THE CONTEXT OF VIDEO

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This discussion forum, an outcome of fruitful collaboration between researchers in the U.S and Germany, focuses on mathematics education research in the context of teaching videos and analyses common findings and their implications for research, teaching and learning of mathematics. We address several important issues such as the use and analysis of videos in conjunction with available theoretical frameworks for (a) the teaching and learning of mathematics, (b) for mathematics teacher education, and (c) for the training of future university educators in the field. Among the various contributions will also include a paper, which identifies culture as a dependent variable in teacher's learning processes. The forum will also be open to a discussion of the use of teaching videos as a research tool. The co-ordinators have gathered teaching videos from the U.S., Germany and the Netherlands to serve as basis for discussion as well as to highlight similarities and differences in mathematics teaching methods across cultures.

Background

Our conception and preference for a particular style of mathematics teaching is the confluence of prior experiences as learners of mathematics. We have a natural tendency to mimic our teachers and adapt teaching practices which are compatible with our beliefs about mathematics and the learning of mathematics. In spite of teachers' best intentions to cultivate a true reform oriented, constructivist classroom where knowledge is constructed via discourse and the give and take of negotiations of meaning, such changes can be quite difficult to achieve. Often there is a tendency to "fall back" on a mental template or script of teaching that is deeply ingrained in our sub-conscious (see Stigler & Hiebert, 2000).

We propose that video can be used to help teachers explore this issue in their own practice and the practice of others. In particular, the purpose of this forum is to stimulate critical debate concerning the use of videos to promote changes in the practice of university educators and teachers and in their conceptions about the teaching and learning of mathematics.

As the limited literature list in this proposal indicates, videos have been used as a medium for teacher training (Sherin & Han, 2004) as well as a research tool in general. The usefulness of videos in research was especially revealed in the video studies conducted in TIMSS, where one could analyze teaching in classrooms in the U.S, Germany and Japan (Stigler & Hiebert, 2000 and the literature quoted). These studies consolidated the fact that mathematics teaching is a culturally based activity, which follow certain scripts which are resistant to alterations. These observations correlate with recent research findings on teacher knowledge and beliefs (e.g.,

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Lloyd, G. M., Wilson, M., Wilkins, J. L. M., & Behm, S. L. (Eds.). (2005).
Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.

Sherin et al., 2000). For example, teacher knowledge has been classified as consisting of two interrelated systems, namely knowledge of lesson structure and content knowledge (Leinhardt & Greeno, 1986; Leinhardt & Smith, 1985). Lesson structure knowledge is described as an understanding of how to plan and implement a lesson, whereas content knowledge consists of understanding the specific mathematics to be taught.

The idea is that these schemata consist of sequences of goals and actions-goals and actions that correspond to what the teacher does in the classroom. Thus, Leinhardt et al. posit a direct relationship between a teacher's knowledge of a lesson and the teacher's behavior in the classroom. Furthermore, Leinhardt and her colleagues do not simply refer to all of these different types of knowledge as schemata; instead, they give names to the various types of schemata, according to the jobs they do and the time scale at which they structure behavior.

There are schemata for:

- mundane activities, such as handing out papers, as well as
- schemata for complex subject matter-specific behavior, such as explaining a difficult concept, repeating standard content, doing calculations (and many more) and
- content-specific schemata (introduction of functions, Pythagoras Theorem, quadratic equations)

Furthermore, these schemata vary greatly in the time-scale at which they structure behavior. While some schemata set a broad plan for a large portion of a classroom session, others are associated with low-level, short duration activities.

According to Schoenfeld (as summarized by Sherin et al (2000)) teaching typically consists of a series of episodes which in turn correspond to a set of actions, termed action sequences. Analogous to zooming into a fractal, each of these episodes are further breakable into more fine tuned action sequences. The underlying skelatal structure of the teaching model manifests when one focuses on these fine tuned action sequences, which include classroom routines, scripts, mini lecturing and talk. Central to the model is the claim that there is a correspondence between these finely tuned actions sequences and classroom goals. Schoenfeld explains that teachers hold multiple goals at multiple grain sizes. Therefore, an action sequence may be related to an overarching goal, a content and/or social goal, as well as more local goals. Similarly, the model elaborates the beliefs and knowledge that influence each action sequence, along with the triggering and terminating events.

In this forum we examine video as a tool for reflection in general, and more specifically, how videos of teaching can elucidate the above model of teaching. Furthermore, we will explore the ways in which understanding teaching as a set of episodes that relate to particular goals and actions can help to promote change both among teachers and among graduate student researchers.

While it is easy to be critical of classroom teachers actively involved in changing their teaching practices and to suggest changes, the university educator must be sensitive to the difficulties and frustrations experienced by teachers during this process. In the U.S., there is a growing trend of doctoral recipients in mathematics education without public school teaching experiences (or even self-reflective teaching experiences), finding themselves in university positions, which invariably involves some training of pre-service and in-service teachers. Yet, many doctoral programs do not involve reflective and critical teaching experiences, necessary to understand the complexities of teaching and the experiences necessary to cultivate the sensitivity required for implementing change in reform oriented classrooms. In the absence of required teaching experiences as requirements in such doctoral programs, it becomes critical that university educators involved in the training of future university educators create the experiences necessary to understand the complexities of teaching. The use of videos in experimental courses on teaching school and college mathematics can greatly aid in this endeavour. Videos are an efficient way of generating data with the caveat that the data although comprehensive in one sense, is also very complex.

Overall Objectives

1. To construct a comprehensive review of the different applications of videos in mathematics teaching.
2. To review the relevance and usefulness of portfolio projects in teacher training.
3. To establish international networks to implement the use of videos for conducting a uniform analysis of teaching in different nations.
4. To explore the use of videos with mathematics education graduate students promote sensitivity about the difficulties of teaching advanced mathematics (Calculus, Abstract Algebra, Analysis) constructively?
5. To classify the types of "mental scripts" that become transparent to graduate students when they critically analyse their teaching videos (a) separately and (b) in collaborative groups.
6. To study the relationship between beliefs about the nature of mathematics and the preference for a particular "underlying" teaching script.
7. To further develop the classification of teaching scripts by generating different examples. For instance classifying seemingly different teaching scenarios which in essence follow one underlying script.
8. To identify beliefs about mathematics which underlie teaching scripts.

It is evident that also technical questions will be addressed (software tools, experiences with these tools, etc.) The discussants have contributed short theoretical reports which will serve as the basis for the discussion. They will also present videos which are part of their research reports. Last not least, we hope to discuss two videos in detail in order to elaborate adequate categories of an analysis.

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BELIEFS ABOUT MATHEMATICS AND TEACHING SCRIPTS FROM THE PERSPECTIVE OF SCHOENFELD'S THEORY OF TEACHING-IN-CONTEXT

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1. Terminology: Following Schoenfeld, we understand beliefs as mental constructs representing the codification of people's experiences and understandings (Schoenfeld, 1998). The problem of identifying mathematical beliefs is well known. The question is how can we actually observe the mental constructs of people? Surveying beliefs via questionnaires only reveals professed beliefs. However there is a major difference between professed beliefs and attributed beliefs. On the other hand, following the theory ‚Teaching-In-Context' (Schoenfeld,

1998) beliefs are an ingredient parameter (among three) to understand the decision-making of a teacher in an actual teaching situation. If one subscribes to the Leinhard Greeno theory that in teaching, certain stable action routines stand out, then we do not completely understand how these action routines found in teaching scripts, are linked with teacher beliefs.

2. An actual teaching situation as starting point: Based on the preceding notes is the analysis of a detailed and limited phase within a video-taped mathematical lesson. The goal of the lesson in grade 8 was to introduce linear functions. The lesson started in an open-ended and problem centered manner, however, in the development of the lesson the teacher changed his/her plan and favored a more classical procedure, namely by communicating definitions as autonomous with the setting of mathematics. The interesting question is what led to this change, which visibly carried itself out as a clear break from the original open-ended delivery of the lesson. One explanation is that due to the over-ambitious agenda of the teacher to deliver the lesson in an open-ended and problem centered manner, time pressure developed which forced the teacher to resort to definitions (safety devices) to meet the end goal of the lesson which was to introduce linear functions. Subsequent interviews confirmed that switching to this classical mathematically structured form of instruction appeared to be the only way out for the instructor to meet the end goal of the lesson. Andelfinger & Voigt (1986) describe a drastically similar procedure in a different context: After an apparent open ended investigative opening of the lesson, the instruction rapidly changed over to the classical form in which the „thing“ to be discovered was told directly to the students as well as ways to procedurally manipulate it. This observation raises several questions:

3. General Observations and Questions: One can suppose that content-specific schemata (introduction of functions, the introduction of the derivative, Pythagoras theorem etc..) from the teacher contain prior basic ideas (concept images). These content-specific schemata for the teaching of complex mathematical ideas are not pedagogically motivated, but contain clues which are traceable to beliefs. Such teacher scripts (with classical safety devices) critically influence the development of the mathematical material in a lesson, whose prior actual goal is to encourage mathematical processes such as argumentation and proof. This radical conversion which occurs during instruction, is inevitably loaded with beliefs about the nature of mathematics.

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VIDEOS IN COLLEGIATE MATHEMATICS TEACHING

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Undergraduate and graduate mathematics courses in the United States are typically taught in the traditional format of direct instruction lecturing. Student understanding of concepts covered in advanced mathematics courses is assessed via timed paper and pencil tests and periodic homework assignments. This traditional delivery of mathematical knowledge with the view of a simple transfer of concepts from teacher to student is based on the Thorndikian premise, that direct instruction plus hard work results in success in advanced mathematics coursework. Yet the attrition rates of numerous minority groups in mathematics, engineering and sciences has been the cause of considerable concern for The Mathematical Association of America (MAA), and resulted in numerous publications (e.g., Gold, 1999; Hibbard & Maycock, 2002; White, 1993; which emphasize alternate methods of teaching and learning in undergraduate and graduate mathematics courses aligned with findings in mathematics education research.

In this report I will discuss the need to emphasize a new approach to the teaching and learning of advanced mathematics with future university mathematicians and mathematics educators enrolled in graduate courses on collegiate mathematics teaching. The objective of one such graduate course taught in Spring 2005 was:

1. to survey in depth recent efforts to reform college mathematics content and teaching.
2. to familiarize students' with special topics within Calculus (Analysis), Abstract Algebra, History of Mathematics, and Statistics.
3. to create "micro" teaching experiences, opportunities for individual reflection on teaching experiences and group critique and analysis of teaching experiences.
4. to familiarize students with frameworks for analyzing collegiate teaching.

Although adopting alternative approaches to the teaching and learning of advanced mathematics was difficult for these students to embrace, the use of current research on collegiate teaching and videos to provide feedback for changing their traditional approach to subjects like Advanced Calculus and Abstract Algebra resulted in a gradual shift from a traditional mode of delivery to a more discovery oriented and humanistic approach to the teaching of these subjects. Student teaching of advanced mathematical topics was video-taped and analyzed (both individually and as a class). Individual reflection and group feedback was used to modify, re-teach and re-re-teach the same topic. Here is a short case summary of the shifts in teaching that occurred with one graduate student (henceforth the "teacher") enrolled in this course. The focus of lesson was on Groups in general. In particular the goal was to introduce the matrix group GLN2 via MATLAB. The first lesson was taught in a very traditional way. That is, the focus of the lesson was on verifying the definition by introducing it at the very beginning and then having students use MATLAB to verify the group axioms for various matrix groups. Several students in the class had difficulty in understanding the various MATLAB commands, which were handled very efficiently by the teacher. One question that was raised was the motivation for computing/generating matrix groups to simply verify the definition as well as its place in mathematics history. Some students were not comfortable with the MATLAB setting and it took considerable amount of time for them to generate the elements of the first two matrix groups and fill out the Cayley table.

Some of the feedback received by the teacher was

(1) To introduce some historical origins and motivation for studying groups.

(2) To have students generate examples of familiar matrix groups to motivate/discover the definition.

(3) To be more involved with what students were doing and increase student-teacher interaction while the lab was being completed.

The second lesson started with a brief excursion into the motivation for using MATLAB to study matrix groups. The lesson began without a foray into outlining the objectives for the particular lesson, and without restating (or having students restate) the definition of a group. In this iteration the class was more comfortable with using MATLAB and students were able to "crank" out the Cayley tables for the first 4 groups. When questioned about the motivation for going through this seemingly procedural exercise, the teacher replied that most extant books on Abstract Algebra only provide "lip-service" to matrix groups and students are left without a good feel for understanding its inherent group structure. This deficiency is remedied by creating a lab where students actually perform a computation via which they "discover" its group structure. The lesson was vastly different from the first lesson because the focus was on discovering the group structure via computation as opposed to simply verifying it. Some of the feedback given to the teacher at the end of the second iteration as:

(1) to delve deeper into the historical origins of matrix groups

(2) to shorten the computations to allow time for class discussion of what had been performed and whether or not a groups structure was visible.

In the second iteration, the teacher was more comfortable in the lab setting, often elicited student comments about what they were performing, and more attentive to students that were still having difficulty with the MATLAB commands.

The third iteration began with a brief outline of the objectives. The machines were already set-up a priori which allowed students to directly jump into the lab activity without going through the routine of getting set up first. The lab activity was considerably shortened which allowed for students to finish the computations. Due the severe time restrictions the teacher was not able to lead a discussion of what the students had done and the results of filling out the Cayley tables. The teacher encouraged students to look up the historical background of matrix groups in general and invoked Heisenberg's work which required the use of matrices to facilitate computations. This got the student's attention. In the three iterations there was a dramatic shift in the focus of the lesson. The shift was measurable in terms of the time allotted to discovering the definition via computation and examples as opposed to the initial (n=1) lesson where the definition motivated the computation simply for verification purposes. The teacher also appeared more relaxed towards the third lesson and there were more incidences of student-teacher interaction. The study shows that systemic change in the teaching of advanced mathematics at the collegiate level is possible via the use of videos in graduate courses on collegiate mathematics teaching.

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UNDERSTANDING THE ROLE OF VIDEO IN TEACHER LEARNING

Miriam Gamoran Sherin

Since video technology became more portable and less costly in the 1960s, numerous activities for teachers have been designed that make use of videotapes of teaching practice. Yet, changes in the ways in which we use video with teachers during this time have not generally been driven by empirical results. Rather than coming about because of an increased understanding of how and why teachers best learn from video, changes have been driven by broader theoretical trends in the educational research community and by advances in technology. This situation is made worse by the lack of theoretical frameworks for describing the process through which teachers learn as they reflect on video.

To address these issues, I explore three themes concerning the role of video in teacher learning. First, I look closely at several key properties of video itself. Specifically, I attempt to identify those features of video that make it particularly useful for teachers (Sherin, 2004). For example, rather than simply use video as a substitute for live classroom observations, I argue that we must consider how video might provide a different perspective than is possible during a live observation. Along these lines, I propose three key affordances of video for teacher education:

(a) video provides a lasting record of classroom interactions; (b) video can be collected and edited; and (c) video affords a different set of practices for teachers. For instance, when watching a pedagogical dilemma on video, one does not have to respond with the immediacy required during teaching. Similarly, video provides teachers with the opportunity to engage in fine-grained analysis of classroom practice, something that is not often possible during the moment of instruction. These affordances have implications both for considering how current video-based programs support teacher learning as well as how future program can effectively leverage what video has to offer teachers.

Second, I examine a particular video-based program called video clubs, professional development cooperatives in which teachers gather to watch and discuss video excerpts of their teaching. Through empirical studies of video clubs, I have sought to understand both what and how teachers learn in this type of setting. For example, Sherin and Han (2004) describe the learning that occurred as four middle-school mathematics teachers participated in a year-long series of video club meetings. Over time, discourse in the video clubs shifted from a primary focus on the teacher to increased attention on students' actions and ideas. In addition, discussions of student thinking moved from simple restatements of students' ideas to detailed analyses of student thinking. Furthermore, teachers began to reframe their discussions of pedagogical issues in terms of student thinking. These types of shifts in the teachers' thinking are particularly important to uncover as they are widely reported to positively influence teachers' ability to implement reform (Ball, 1993; Franke et al., 2001).

Third, my analyses of teacher learning via video focus on the notion of teachers' professional vision (Goodwin, 1994; Sherin, 2001; van Es & Sherin, 2002). The idea is that professionals in a discipline develop specialized knowledge in order to interpret the phenomena of interest to them. Thus, archeologists have professional vision for examining sand and stones, and meteorologists have professional vision for examining the shape and movement of weather systems. Because the phenomena of interest to teachers are classrooms, we can think of teachers as having professional vision for identifying and interpreting significant features of classroom interactions. I believe that much of the important teacher learning that occurs during video clubs can be understood in terms of changes to the teachers' professional vision. Specifically, as discussed above, I found that, over time, the teachers began to notice new aspects of classroom interactions. They also developed new strategies for making sense of what they noticed. Furthermore, there was a dynamic interaction between the teachers' noticing of key events and their interpretations of these events. Thus, what teachers noticed influenced their interpretative strategies, but in addition, as they developed more sophisticated interpretive strategies, what they noticed became more complex. Future research will examine the extent to which a focus on professional vision provides a way to characterize key components of teacher learning in other kinds of video-based programs for teachers.

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OBSERVING THE PROVING PROCESS IN MATHEMATICS CLASSROOM

Aiso Heinze

In the last years several studies on the proving process in the mathematics classroom were conducted (e.g. Herbst, 2002; Heinze, 2004). These studies analysed the question how mathematical proof is taught in school from different methodological perspectives.

The investigation of Herbst (2002) revealed different kind of problems the teachers are faced with in the mathematics lessons when they are organising the proofs in a two-column format. Herbst (2002) identified a double bind on the teacher, because, on the hand, she/he has to create a learning situation that is based on certain didactical ideas to facilitate the content for the students. On the other hand, by this kind of teacher support the students are obliged to understand the didactical situation and to follow the prepared way the teacher has planned. The consequence is that the teacher cannot identify if student problems in the lessons are based on an inadequate mathematical understanding or on an inadequate understanding of the didactical situation.

The situation in Germany is somewhat different, because there is no strong classroom tradition regarding the two-column format. However, in Germany we find a tradition regarding a general teaching style for mathematics lessons which is called the "fragend-entwickelnde" teaching style. It is a kind of classroom discourse, in which the teacher tries to develop the content by asking questions to the students. The interaction between teacher and students is framed by the social norms of this discourse. There are several studies on this teaching style which identified certain scripts like the funnel-shaping of the teacher questions, i.e. if the students' answers are not suitable to the questions the teacher will more and more close the question till the answer is more or less trivial.

In a video study based on 20 lessons on geometry proofs in grade 8 we analysed how the proving process in the German mathematics classroom is organised (Heinze, 2004). As a basis for this investigation we took a model of the proving process which consists of five phases: (1) finding of the hypotheses, (2) formulation of the hypothesis, (3) exploring the hypothesis and generate a proof idea, (4) creation of a sketch of the proof, (5) formulation of the proof and retrospective summary. In contrast to the proving process in academic mathematics, which is divided in a private and a public part, in the geometry classroom we found hardly any private parts in the proving processes. In general, the teacher asks the students to make a geometrical construction and to measure some angles and lines. The results were collected and a hypothesis was derived. After that the students got shortly the possibility to give some ideas for a proof. These ideas were in general not successful, because they were spontaneous and not based on a deeper investigation of the problem. Then within the "fragend-entwickelnde" teaching framework the teacher worked out the proof idea and the final proof step by step at the black board.

Our results indicate that particularly the exploration phase in the proving process is neglected by the teachers. In the observed proof lessons the students were hardly able to get any idea of how to explore a proof task systematically and how to generate a first approach to a proof. The consequences are that students get an inadequate understanding of mathematical proofs and lack of strategies to solve proof problems.

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CROSS-CULTURAL ELEMENTS OF LESSON STRUCTURE IN CLASSROOMS FROM GERMANY, HONG KONG AND THE USA

Eva Jablonka

The findings reported in this section rest upon six case studies that used the data produced in the Learner's Perspective Study (LPS). The LPS studies the practices and associated meanings in eighth-grade mathematics classrooms in 13 countries (see http://extranet.edfac.unimelb.edu.au/DSME/lps/). Each county participating in the LPS used the same research design to collect videotaped classroom data for ten consecutive mathematics lessons and to conduct post-lesson video-stimulated interviews with two students after each lesson in each of three participating 8th grade classrooms. In part, the LPS study is motivated by the postulated cultural specificity of teacher practice and by the belief that the characterization of the practices of a mathematics classroom must attend to learner practice with at least the same priority as that accorded to teacher practice. The methodology of data production in the LPS aims at documenting not just the obvious events that might be recorded on videotape, but also the participants' construal of those events.

One rationale for studying lesson structure internationally is to identify elements of lesson plans and classroom interaction that grew out of local traditions of curriculum and teacher training and are evidenced in distinct organizational forms. It can be assumed that the availability of a repertoire of specific forms of activities shapes the process of learning/teaching.

On the other hand, it is interesting to see how rather cross-national elements of mathematics lessons turn out to be enacted in distinct ways in different classrooms within and across national traditions. Such a study could offer insights into the extent to which these forms are shaped by similarities of teachers' and students' perceptions of mathematics and its teaching/learning.

For example, in all six classrooms from this study, teachers frequently employ a form of interaction and talk that consists of addressing the whole class by a series of connected questions; the students volunteer for getting a turn by raising their hands; the teacher selects the turns, opens and closes the discussion and evaluates the contributions of the students. The questions do not aim at eliciting information from the students that is not accessible to the teacher, but at eliciting information from the students in order to incorporate it into a collective development of the topic. Microanalyses of these episodes show how students adapt to this form of interaction.

Searching for cross-cultural similarities in mathematics classrooms can help to identify different ways of how teachers and students deal with the same problems and which modes of teaching/learning constrain or afford students' learning.