MODELS AND MODELING WORKING GROUP

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The Models and Modeling Working Group has provided participants with a setting to reflect on models and modeling perspectives to understand how students and teachers learn and reason about real life situations encountered in a mathematics and science classroom. From these perspectives, a model is considered as a conceptual system that is expressed by using external representational media, and that is used to construct, describe, or explain the behaviors of other systems. There are different types of models that students and teachers develop (explicitly) to construct, describe, or explain mathematically significant systems that they encounter in their everyday experiences, as these models are elicited through the use of model-eliciting activities (Lesh, Hoover, Hole, Kelly, & Post, 2000). During the workshop we will continue to explore these aspects of learning, teaching, and research.

New directions for Models and Modeling Perspectives will be the topic of discussion and dissemination during this year's Working Group. Current and important achievements include collaborative work that has been done in innovative research design and assessment. These attainments include the soon release of several publications that will focus on Real-World Models and Modeling as a Foundation for the Future of Mathematics Education, Design-based Research, and Assessment Design. These three will be the main themes addressed during this year's Working Group.

Introduction

The Models and Modeling Working Group at PME-NA XVII has the following goals:

- To disseminate and contribute to the research on the use of models and modeling in school mathematics, with a focus on students, teachers, researchers, and policy makers.
- To create and support collaborations among researchers to build international communities of practice.
- To extend the field of mathematics education towards new directions on assessment, problem solving, research design, learning environments and complexity; as it relates to the use of models and modeling in school mathematics.

Highlights of a Models and Modeling Perspective

The Models and Modeling Working Group has provided participants with a setting to reflect on *models and modeling perspectives* to understand how students and teachers learn and reason about real life situations encountered in a mathematics and science classroom. From these perspectives, a model is considered as a conceptual system that is expressed by using external representational media, and that is used to construct, describe, or explain the behaviors of other systems. There are different types of models that students and teachers develop (explicitly) to construct, describe, or explain mathematically significant systems that they encounter in their everyday experiences, as these models are elicited through the use of model-eliciting activities

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(Lesh, Hoover, Hole, Kelly, & Post, 2000). During the workshop we will continue to explore these aspects of learning, teaching, and research.

For several years, the Models and Modeling Working Group at PME and PME-NA has been a productive setting for participants to present results, develop new ideas, and create new directions for the use of models and modeling in school mathematics. In 2003, the book *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Problem Solving, Learning, and Teaching*, edited by Richard Lesh and Helen Doerr, crystallized many of the outcomes and collaborations that emerged from participants in this Working Group.

Models and modeling perspectives have proved to be a rich context for research and development. One of the main points of convergence from the conclusions achieved by each of the groups in the past workshops resided on the need for innovative designs for research and assessment that can help answer questions that involve the understanding of complex situations that are dynamic and iterative, like the ones needed to study advances in: Student Development (Aliprantis & Carmona, 2003; Johnson & Lesh, 2003; Kardos, 2003; Dark, 2003; Lehrer & Schauble, 2003; Lesh & Carmona, 2003; Lesh, Zawojewski, & Carmona, 2003; Oakes & Rud, 2003), Teacher Development (Hjalmarson, 2004; Doerr & Lesh, 2003; Koellner Clark & Lesh, 2003; McClain, 2003; Schorr & Lesh, 2003; van Reeuwijk & Wijers, 2003), Assessment (Trigueros & Carmona, 2005a, 2005b; Carmona, 2004; Lesh & Lamon, 1993), Curriculum Development (Cramer, 2003; Carlson, Larsen, & Lesh, 2003; Lamon, 2003; Lesh, Cramer, Doerr, Post, & Zawojewski, 2003; Shternberg & Yerushalmy, 2003), and Problem Solving (English & Lesh, 2003; Harel & Lesh, 2003; Kehle & Lester, 2003; Lesh, Lester, & Hjalmarson, 2003; Lester & Kehle, 2003; Middleton, Lesh, & Heger, 2003; Zawojewski & Lesh, 2003; Zawojewski, Lesh, & English, 2003).

New directions for Models and Modeling Perspectives will be the topic of discussion and dissemination during this year's Working Group. Current and important achievements include collaborative work that has been done in innovative research design and assessment. These attainments include the soon release of several publications that will focus on *Real-World Models and Modeling as a Foundation for the Future of Mathematics Education*, Design-based Research, and Assessment Design. These three will be the main themes addressed during this year's Working Group.

Real-World Models and Modeling as a Foundation for the Future of Mathematics Education

Discussion and presentations on *Real-World Models and Modeling as a Foundation for the Future of Mathematics Education* will be guided by the following questions: *How can research investigate systems of interacting systems –in situations where students interact with one another, students interact with teachers and students, teachers interact within continually evolving learning communities, and the learning activities are themselves continually evolving situations? What steps can be taken to develop a research community that is more than just a community of isolated individuals?*

During panel discussions and presentations, emphasis will be made on the research needs in mathematics education; and how fields like engineering or other design sciences can help inform research methods and models to explore and better understand the types of settings encountered in our field. In particular, participants will be able to examine the distinction between single-theory-based research and problem-based research. Single-theory-based research starts with a theory and seeks to establish principles within it –usually by testing hypotheses in "real world"

situations. In contrast, problem-based research starts with a problem that needs to be solved, or an artifact that needs to be designed; and the results typically draw on more than a single theory. "Real life" problems often involve too much and not enough information –as well as too little time, too few resources, and conflicting goals (such as low costs versus high quality, time-efficient versus competence, to mention a few).

Most of the systems that are priorities for math educators to understand and explain are complex systems; and one of the distinguishing characteristics of mathematically complex systems is that the system-as-a-whole have "emergent properties" which cannot be deduced from properties of isolated elements of the systems. These "emergent properties" cannot be described by using single-function models –or even using lists of single function models. Rather, these complex systems are more similar to the problem-based systems that engineers need to understand and explain –such as: complex programs of instruction, interacting with complex learning activities, in which complex conceptual systems of students, teachers, and researchers are functioning, interacting, and adapting. Within such a systems, feedback loops and systems-as-a-whole develop patterns and properties in which results among elements of the systems cannot be derived or deduced from an aggregation or collection of individual properties of elements, or from properties of elements themselves plus properties of any "treatment" that might be used.

Design-Based Research in Mathematics Education

Appropriate research methodologies need to be explored and used in order to study and better understand educational phenomena as complex systems of the sort previously described. The book *Design Research in Education* (Kelly & Lesh, in press), a sequel to the earlier *Handbook of Research Design in Mathematics and Science Education* (Kelly & Lesh, 2000), is a significant and innovative effort to explicate, adopt, and extend the use of this type of research design from fields like engineering and technology, to the field of education; with an overall goal of understanding and improving teaching and learning.

Based on the term and the characteristics described by Brown (1992), Collins (1992), and Collins, Joseph & Bielaczyc (2004), we will call such research design a "design experiment" or design-based research. This type of research design can be characterized through four general principles, which focus on the development of constructs and conceptual systems used by students, teachers, or researchers.

1. The Externalization Principle

Situations should be identified in which the relevant ways of thinking that are desired to investigate (and/or develop) are expressed in forms that are visible to both researchers and to relevant participants. Design activities naturally tend to lead to *thought-revealing artifacts*, like the model-eliciting activities (Lesh, Hoover, Hole, Kelly, & Post, 2000). For these activities, the underlying design often is apparent in things that are designed; the underlying constructs often are apparent in complex artifacts that are constructed; and, the underlying models often are apparent in conceptual tools that embody them. In other words, in the process of designing complex artifacts and conceptual tools, participants often externalize their current ways of thinking in forms that reveal the constructs and conceptual systems that are employed. Therefore, as the tools or artifacts are tested, revised, or refined, the underlying ways of thinking are also tested, revised, and refined; and these cycles are made visible, leaving trails of documentation of the designer's development over time.

2. The Self-Assessment Principle

Design "specs" should be specified as criteria that can be used to test and revise trial artifacts and conceptual tools (as well as underlying ways of thinking) –while discerning products that are unacceptable, or that are less acceptable than others. The design "specs" should function as Dewey-style "ends-in-view". That is, they should provide criteria so that formative feedback and consensus building can be used to refine thinking in ways that are progressively "better" based on judgments that can be made by participants themselves. In particular, ends-in-view should enable participants to make their own judgments about: (a) the need to go beyond their first primitive ways of thinking, and (b) the relative strengths and weaknesses of alternative ways of thinking that emerge during the design process. Productive ends-in-view also should require participants to develop constructs and conceptual systems that are: (a) powerful (to meet the needs of the client in the specific situation at hand), (b) shareable (with other people), (c) re-usable (for other purposes), and (d) transportable (to other situations). In other words, both the tools and the underlying ways of thinking should be shareable and generalizable.

3. The Multiple Design Cycle Principle (or the Knowledge Accumulation Principle)

Design processes should be used in such a way that participants clearly understand that a series of iterative design cycles are likely to be needed in order to produce results that are sufficiently powerful and useful. If design processes involve a series of iterative development>testing>revision cycles, and if intermediate results are expressed in forms that can be examined by outside observers as well as by the participants themselves, then auditable trails of documentation are generated automatically; and, this documentation should reveal important characteristics of developments that occur. In other words, the design processes should contribute to learning as well as to the documentation and assessment of learning.

4. The Diversity and Triangulation Principle

Design processes should promote interactions among participants who have diverse perspectives; and, they also should involve iterative consensus building –to ensure that the knowledge, tools, and artifacts will be shareable and reusable- and so that knowledge accumulates in ways that build iteratively on what was learned during past experiences and previous design cycles. In general, to develop complex artifacts and tools, it is productive for participants to work in small groups consisting of 3-5 individuals who have diverse understandings, abilities, experiences, and agendas. By working in such groups, communities of relevant constructs tend to emerge in which participants need to communicate their current ways of thinking in forms that are accessible to others. Once diverse ways of thinking emerge, selection processes should include not only feedback based on how the tools and artifacts work according to the ends-in-view that were specified –but also according to feedback based on peer review. In this way, consensus-building processes involve triangulation that is based on multiple perspectives and interpretations. So, the collective constructs that develop are designed to be shareable among members of the group; and, they are designed in ways so that knowledge accumulates.

In addition, design-based research should maintain the following goals. First, to radically increase the relevance of research to practice –often by involving many levels and types of practitioners in the identification and formulation of problems to be addressed- or in the interpretation of results, or in other key roles in the research process. Second, to acknowledge that most of the things that need to be understood and explained in mathematics education are complex systems –not necessarily in the strict mathematical sense, but at least in the general

sense that they are dynamic, interacting, self-regulating, and continually adapting. Third, to recognize that the mathematical models that are needed to describe and explain the preceding systems are not restricted to linear equations or other kinds of simple input-output rules that presuppose the existence of independent variables that can be isolated, factored out, or controlled (Lesh & Lamon, 1992). Fourth, to acknowledge that research is about knowledge development; and, that not all knowledge is reducible to a list of tested hypotheses and answered questions. In particular, in mathematics and science education, the products that require emerging new research designs are intended to emphasize the development of models (or other types of conceptual tools) for construction, description, or explanation of complex systems. When producing these latter types of products, distinctions are being made between: (a) model development studies and model testing studies, (b) hypothesis generating studies and hypothesis testing studies, and (c) studies aimed at identifying productive questions versus those aimed at answering questions that practitioners already consider to be priorities.

Some of these questions that will lead to the re-forming and informing the nature of how innovations, infrastructure and implementations in education should be constructed; and which will be approached during this year's Models and Modeling Working Group include: *What are appropriate models to understand the behavior of complex, self-organizing systems? How can we design powerful artifacts that will help in this modeling process? How can we gather information that will advise the development of a science of learning? What are appropriate methods for data collection of a design study, so that other communities can learn from and be persuaded by the study (including researchers, teachers, administrators, parents, students, and policy-makers)? What constitutes evidence that an intervention needs refinement or revision? How can we conduct educational research on innovative practices using novel technologies that can be viewed as accumulating science and providing scalable artifacts to positively influence future practice?*

Models and Modeling in Assessment Design

Similar to the previous discussion about the need to make a shift on the types of models and research design to better reflect the current changes and development in social sciences, *models and modeling perspectives* present the need to shift views in the development of assessment designs that more accurately reflect the types of knowledge development in students and teachers. This will be the third focus for discussion in the Models and Modeling Working Group during this year's PME-NA. Presentations and discussion panels will describe new types of dynamic and iterative assessments that are especially useful in design research –where rapid multi-dimensional feedback is needed about the behaviors of complex, dynamic, interacting, and continually adapting systems.

Most of the work being done in assessment includes the production of tests, whose outcome is a number – instead of thinking about knowledge development, and how it can be improved. Assessment needs to be more complex than explaining knowledge development by just producing a number. A focus shift needs to be made in Assessment Design towards producing conceptual tools that provide useful information for decision makers.

From *models and modeling perspectives*, a more appropriate Assessment Design is such that incorporates a holistic view of the education, considering it as a complex system, as previously described (where students, teachers, policy makers, and other educators interact; changing dynamically, modifying and being modified by the curriculum, assessment, classroom

environment, and other factors). Among the things that need to be considered in the Assessment Design, and that will be part of the discussion include the following questions:

1. Focus on the decision-makers. Different decision-makers will need and value different information in order to make decisions. Thus, it is important to identify: *Who will be the decision makers? Teachers? Policy makers? Parents? Tax Payers? Administrators? Students? Learning Communities?*
2. Establishing Goals. *What decisions are priorities for these decision makers? For what purposes? What are their ends-in-view?*
3. Operational Definitions. *What is it that needs to be described, assessed, or measured? What is understood by "good" and "better"? Under what circumstances? What attributes should be valued? How can relevant conceptual systems be understood without partitioning into meaningless pieces to be measured, but providing a holistic and systemic view?*
4. Designing Tools. *What conceptual tools should be designed in order to document participants' development and interactions? How can models be designed so that knowledge is documented at the same time as it is being developed?*

The Assessment Design that will be considered from *models and modeling perspectives* is such that allows for the different participants (students, teachers, researchers, policy makers, to mention a few) to design artifacts or models that document their knowledge development at the same time as it is being created. This documentation should also elicit the interactions among different participants, and how these interactions contribute to the development of the whole system (Lesh & Kelly, 2000). In addition, following advances and new developments in technology and design, it is no longer necessary for educational decision-makers to rely on reports that involve nothing more than simple-minded unidimensional reductions of the complex systems that characterize the thinking of students or teachers –or relevant communities; rather, capabilities exist to use graphic, dynamic, and interactive multimedia displays to generate simple (but not simple minded) descriptions of complex systems (similar to weather systems, traffic patterns, biological systems, dynamic and rapidly evolving economic systems, to mention a few examples). Thus, new reporting artifacts should be developed that allow a better depiction, assessment, and evaluation of the phenomena.

The Working Group at PME-NA XXVII

For the PME-NA XXVII Models and Modeling Working Group, several sessions will be organized throughout the Conference. In particular, there will be two main working group sessions. For each session, after a general introduction on different topics is provided, participants will be invited to select one, and smaller groups will be formed. Each sub-group will have a panel of discussants, and a discussion leader, who will approach the selected theme. In addition, participants will be encouraged to attend to other sessions that will be offered throughout the Conference, and that will further support and enrich the discussion that will take place during the two Working Group sessions.

The topics that will be discussed during the Working Group Sessions are:

Working Group Session 1

Discussion Group Topics:

- Modeling Students' Modeling Abilities – New Directions for Research Collaborations
  Panel: Les Steffe, Tom Kieren, Tom Post, Jeremy Roschelle
  Richard Lesh (discussion leader)

- Modeling Perspectives on New Directions for Research on Problem Solving?
  Panel: Judi Zawojewski, Caroline Yoon, Frank Lester, Eric Hamilton
  Margret Hjalmarson (discussion leader)
- Modeling Perspectives on Design Research Methodologies for Assessing Complex Achievements
  Panel: Eamonn Kelly, Finbarr Sloane, Lyn English, Roberta Schorr
  Lupita Carmona (discussion leader)
- Modeling Perspectives – New Directions for Research on Teacher's Knowledge
  Pat Thompson, John Mason, Kay McClain
  Helen Doerr (discussion leader)

Working Group Session 2

Discussion Group Topics:

- New Directions for Research in the Primary Grades
  Panel: Lyn English, Shweta Gupta, Jennifer Fonseca
  Joan Moss (discussion leader)
- New Directions for Research at the University Level
  Panel: Eric Hamilton, Judi Zawojewski, Sally Berenson, Marilyn Carlson
  Maria Droujkova (discussion leader)
- Learning Environments where the Problem Solver is a Group
  Panel: Jim Kaput (Andy Hurford), Walter Stroup, Eric Hamilton
  Jim Middleton (discussion leader)
- Modeling Perspectives on What's Needed for Success beyond School?
  Panel: Jim Kaput (Jeremy Roschelle), Jerry Goldin, Pat Thompson
  Richard Lesh (discussion leader)

The additional sessions include six discussion panels on the following topics:

Panel 1: Modeling Students' Modeling Abilities – New Directions for Multi-Site Research Collaborations
Les Steffe, Tom Kieren, Jeremy Roschelle
Richard Lesh (discussion leader)

Panel 2: Modeling Perspectives on New Directions for Research on Problem Solving?
Judi Zawojewski, Caroline Yoon, Frank Lester, Eric Hamilton
Margret Hjalmarson (discussion leader)

Panel 3: Modeling Perspectives on New Directions for Research on Teacher's Knowledge
Pat Thompson, John Mason, Kay McClain
Helen Doerr (discussion leader)

Panel 4: Modeling Perspectives – What's Needed for Success beyond School?
Jim Kaput (Walter Stroup), Jerry Goldin, Eric Hamilton
Richard Lesh (session coordinator)

Panel 5: Modeling Perspectives on Design Research Methodologies to Investigate or Assess Complex Achievements
Eamonn Kelly, Finbarr Sloane, Lyn English, Roberta Schorr
Lupita Carmona (discussion leader)

Panel 6: Modeling Perspectives on Students' Developing Mathematical Knowledge
Jim Middleton, Marilyn Carlson, Tom Kieren, Les Steffe
"Tom Post (discussion leader)

References

Aliprantis, C. D. & Carmona, G. (2003). Introduction to an economic problem: a models and modeling perspective. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Brown, A.L. (1992). Design experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *Journal of the Learning Sciences*, 2(2), 141-178.

Carlson, M., Larsen, S., & Lesh, R. A. (2003). Integrating a models and modeling perspective with existing research and practice. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Carmona, G. (2004). *Designing an assessment tool to describe students' mathematical knowledge*. Ph.D. Thesis. West Lafayette, IN: Purdue University.

Collins, A. (1992). Toward a Design Science of Education. In E. Scanlon and T. O'Shea (eds.), *New Directions in Educational Technology*. New York: Springer-Verlag.

Collins, A., Joseph, D., & Bielaczyc, K. (2004). Design research: Theoretical and methodological issues. *Journal of the Learning Sciences, 13*(1), 15-42.

Cramer, K. (2003). Using a translation model for curriculum development and classroom instruction. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Dark, M. J. (2003). A models and modeling perspective on skills for the high performance workplace. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Doerr, H. & Lesh, R. A. (2003). A modeling perspective on teacher development. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. A. & Doerr, H.M. (2003). *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Teaching, Learning, and Problem Solving*. Mahawah, N.J.: Lawrence Erlbaum.

English, L. & Lesh, R. A. (2003). Ends-in-view problems. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Harel, G. & Lesh, R. (2003). Local conceptual development of proof schemes in a cooperative learning setting. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Hjalmarson, M. (2004). *Designing presentation tools: A window into teacher practice*. Ph.D. Thesis. West Lafayette, IN: Purdue University.

Johnson, T. & Lesh, R. A. (2003). A models and modeling perspective on technology-based representational media. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Kardos, G. (2003). The case of cases. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Kehle, P. E. & Lester, F. K. (2003). A semiotic look at modeling behavior. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Kelly, A. E., & Lesh, R. A. (2000). *Handbook of research design in mathematics and science education*. Mahwah, N.J.: Lawrence Erlbaum.

Kelly, A. E., & Lesh, R. A. (in press). *Design research in education*. Mahwah, N.J.: Lawrence Erlbaum.

Koellner Clark, K. & Lesh, R. A. (2003). A modeling approach to describe teacher knowledge. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lamon, S. (2003). Beyond constructivism: an improved fitness methaphor for the acquisition of mathematical knowledge. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lehrer, R. & Schauble, L. (2003). Origins and evolutions of model-based reasoning in mathematics and science. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. A. & Carmona, G. (2003). Piagetian conceptual systems and models for mathematizing everyday experiences. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. A. & Doerr, H.M. (2003). *Beyond Constructivism: Models and Modeling Perspectives on Mathematics Teaching, Learning, and Problem Solving*. Mahawah, N.J.: Lawrence Erlbaum.

Lesh, R., Hoover, M., Hole, B., Kelly, A., & Post, T. (2000). Principles for Developing Thought-Revealing Activties for Students and Teachers. In Lesh, R. & Kelly, A. (Eds.) *Handbook of Research Design in Mathematics and Science Education*. Hillsdale, NJ: Lawrence Erlbaum Associates, Inc.

Lesh, R. A., & Kelly, E. A. (2000). Multitiered teaching experiments. In E. A. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education*. Mahawah, NJ: Lawrence Erlbaum.

Lesh, R. A., & Lamon, S. J. (1992). *Assessment of authentic performance in school mathematics*. Washington, D.C.: American Association for the Advancement of Science.

Lesh, R. A., Lester, F. K., & Hjalmarson, M. (2003). A models and modeling perspective on metacognitive functioning in everyday situations where problem solvers develop mathematical constructs. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models*

and modeling perspectives on mathematics problem solving, learning, and teaching. Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. A., Zawojewski, J. S., Carmona, G. (2003). What mathematical abilities are needed for success beyond school in a technology-based age of information? In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Lester, F. K. & Kehle, P. (2003). From problem solving to modeling: the evolution of thinking about research on complex mathematical activity. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

McClain, K. (2003). Task-analysis cycles as tools for supporting students' mathematical development. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Middleton, J. A., Lesh, R. A., & Heger, M. (2003). Interest, identity, and social functioning: central features of modeling activity. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Oakes, W. & Rud, A. G. (2003). The EPICS model in engineering education: perspective on problem-solving abilities needed for success beyond schools. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Schorr, R. & Lesh, R. A. (2003). A modeling approach for providing teacher development. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Shternberg, B. & Yerushalmy, M. (2003). Models of functions and models of situations: on the design of modeling-based learning environments. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Trigueros, M. & Carmona, G. (2005). Evaluación del Programa Piloto. In T. Rojano (Ed.) *Nuevas tecnologías en la enseñanza de las ciencias y las matemáticas en secundaria*. Organización de Estados Iberoamericanos.

Trigueros, M. & Carmona, G. (2005). Nuevas Perspectivas de Evaluación. In T. Rojano (Ed.) *Nuevas tecnologías en la enseñanza de las ciencias y las matemáticas en secundaria*. Organización de Estados Iberoamericanos.

Van Reeuwijk, M. & Wijers, M. (2003). Explanations why? The role of explanations in answers to (assessment) problems. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Zawojewski, J. S. & Lesh, R. A. (2003). A models and modeling perspective on problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching*. Mahwah, NJ: Lawrence Erlbaum Associates.

Zawojewski, J. S., Lesh, R. A., & English, L. (2003). A models and modeling perspective on the role of small group learning activities. In R. Lesh & H. M. Doerr (Eds.), *Beyond*

constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching. Mahwah, NJ: Lawrence Erlbaum Associates.