Theoretical Results on Base Station Movement Problem for Sensor Network

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***Abstract*—The benefits of using mobile base station to prolong sensor network lifetime have been well recognized. However, due to the complexity of the problem (time-dependent network topology and traffic routing), theoretical performance limit and provably optimal algorithms remain difficult to develop. This paper fills this important gap by contributing theoretical results regarding the optimal movement of a mobile base station. Our main result hinges upon a novel transformation of the joint base station movement and flow routing problem from time domain to space domain. Based on this transformation, we first show that if the base station is allowed to be present only on a set of pre-defined points, then we can find the optimal time span for the base station on each of these points so that the overall network lifetime is maximized. Based on this finding, we show that when the location of the base station is un-constrained (i.e., can move to any point in the two-dimensional plane), we can develop an approximation algorithm for the joint mobile base station location and flow routing problem such that the network lifetime is guaranteed to be at least $(1-\varepsilon)$ of the maximum network lifetime, where ε can be made arbitrarily small depending on required precision.**

I. Introduction

The benefits of using mobile base station to prolong sensor network lifetime have been well recognized [8], [20]. Since base station is the sink node for data collected by all the sensor nodes in the network, this approach aims to alleviate the traffic aggregation burden from a fixed set of sensor nodes near the base station to other sensor nodes in the network, and it is possible to extend the network lifetime significantly.

Although the potential benefit of using mobile base station to prolong sensor network lifetime is significant, the theoretical difficulty of this problem is enormous. There are two components that are tightly coupled in this problem. First, the location of the base station is now a function of time, i.e., at different time instances, we have a different physical network topology, with the sink node being at different positions. Second, the multi-hop traffic (or flow) routing behavior may change with both time as well as the location of the base station. As a result, an optimization problem with the objective of maximizing the network lifetime needs to consider both base station location (time-dependent) and flow routing. Due to these difficulties, existing solutions to this problem remain heuristic at best (e.g., [8], [20]) and cannot provide *provably* optimal solution to network lifetime performance.

To fill in this theoretical gap, this paper offers an in-depth study on the network lifetime performance limit when mobile base station is employed. We formulate an optimization problem with base station movement and multi-hop flow routing as part of the constraints. As a first step, we show that as far as network lifetime objective is concerned, flow routing is only dependent on the base station location, regardless of *when* the base station should be present at this location. Further, the specific time instances for the base station to visit a location is not important, as long as the total time span for the base station to be present at this location is the same. With this finding, we show how to make a novel transformation from a time-dependent problem formulation to location (space)-dependent problem formulation.

As a second step, we show that when the base station is only allowed to be present at a finite set of pre-determined points (called *constrained mobile base station* (C-MB) problem), we can find the optimal time span for the base station to stay on each of these points (as well as the corresponding flow routing solution) such that the overall network lifetime (i.e., sum of the time spans) is maximized via a single linear program (LP).

Building upon these results, the main result in this paper (Section V) shows that for the *un-constrained mobile base station* (U-MB) problem, i.e., the base station can be present at any point in the two dimensional plane, we can develop a *provably* $(1-\varepsilon)$ optimal algorithm that provides a solution with network lifetime guaranteed to be at least $(1-\varepsilon)$ of the maximum network lifetime (albeit it is unknown), where ε can be made arbitrarily small depending on required precision. The main idea in this approximation algorithm is to exploit a clever way of dividing the search space into subareas, with each of its link cost having some nice properties that are related to $(1-\varepsilon)$ optimal. A novel idea in the design of $(1-\varepsilon)$ optimal algorithm is to represent each subarea with so-called "fictitious cost point," which is an N-tuple cost vector with each component representing an upper bound of cost to the respective node. As a result, we can apply the LP approach developed for the C-MB problem on the fictitious cost points and develop provably $(1-\varepsilon)$ optimal solution.

The rest of this paper is organized as follows. In Section II, we describe the network model and introduce the mobile base station problem. In Section III, we present a novel transformation that enables to transform a time-dependent problem to a space-dependent problem. In Section IV, we develop optimal solution for the constrained mobile base station (C-MB) problem. Section V presents an algorithm for

TABLE I
NOTATION.

General notation	
$\mathcal{N}$	The set of sensor nodes in the network
N	$=\lvert\mathcal{N}\rvert$, the number of sensor nodes in the network
B	Denotes the base station
$(x,y)(t)$	Location of base station B at time t
(x_i,y_i)	Location of sensor node i
r_i	Bit rate generated at sensor node i
e_i	Initial energy at sensor node i
α,β	Two constant terms in power consumption model for data transmission
ρ	Power consumption coefficient for receiving data
n	Path loss index
C_{ij}	Cost for transmitting data from sensor i to sensor j
$c_{iB}(t)$	Cost for transmitting data from sensor i to base station B at time t
$c_{iB}(p)$	Cost for transmitting data from sensor node i to base station B when B is at point p
$g_{ij}(t)$ (or $g_{iB}(t)$)	Flow rate from sensor node i to sensor node j (or base station B) at time t
$f_{ij}(p)$ (or $f_{iB}(p)$)	Flow rate from sensor node i to sensor node j (or base station B) when B is at point p
$W(p)$	Time span for the base station to be present at point p
C-MB problem specific notation	
M	The number of pre-determined locations
$\psi^*_{\text{C-MB}}$	An optimal solution to the C-MB problem
$T^*_{\text{C-MB}}$	The maximum network lifetime achieved by $\psi^*_{\text{C-MB}}$
U-MB problem specific notation	
ε	Required approximation precision, $\varepsilon > 0$ and $\varepsilon \ll 1$
$\mathcal{A}$	The search space for the base station, which can be the smallest enclosing disk to cover all sensor nodes
$O_{\mathcal{A}}, R_{\mathcal{A}}$	The center and radius of $\mathcal{A}$
M	The number of subareas
$\mathcal{A}_m$	The m-th subarea in the search space
$W(A_m)$	Time span for the base station to be present in subarea A_m
$C^{\min}_{iB}, C^{\max}_{iB}$	Lower and upper bounds of $c_{iB}(p)$
$C[h]$	$=\alpha(1+\varepsilon)^h$, the transmission cost for the h-th circle
H_i	The required number of circles at sensor node i
$\psi_{\text{U-MB}}$	$(1-\varepsilon)$ optimal solution to the U-MB problem
$T_{\text{U-MB}}$	$(1-\varepsilon)$ optimal network lifetime achieved by $\psi_{\text{U-MB}}$

the unconstrained mobile base station (U-MB) problem with provably $(1-\varepsilon)$ optimal network lifetime. Section VI reviews related work and Section VII concludes this paper.

II. Network Model and Problem Formulation

A. Network Model

We consider a set of sensor nodes $\mathcal{N}$ deployed over a two-dimensional area, with the location of each sensor node $i \in \mathcal{N}$ being at a point (x_i, y_i) . We assume each node generates data at a rate of r_i. There is a base station B for the sensor network and it serves as the sink node for all data collected by the sensor nodes. Data generated by each sensor node can be relayed via single or multi-hop toward the base station.

We now discuss the energy consumption due to communications (i.e., data transmission and reception). We assume that each node has power control capability. Suppose that node i transmits data to node j with a rate of f_{ij}, then the transmission power at node i can be modeled as [7]

$$p^t_{ij} = C_{ij} \cdot f_{ij} \ , \tag{1}$$

where C_{ij} is the cost per bit rate between nodes i and j and can be modeled as

$$C_{ij} = \alpha + \beta \cdot d^n_{ij} \ , \tag{2}$$

where α and β are two constant terms, d_{ij} is the physical distance between nodes i and j, and n is the path loss index and is typically with $2 \le n \le 4$ [11].

The receiving power consumption at sensor node i can be modeled as [7]

$$p^r_i = \rho \sum_{k \in \mathcal{N}}^{k \neq i} f_{ki} \ , \tag{3}$$

where ρ is a constant and f_{ki} is the incoming bit-rate received by sensor node i from sensor node k.

In this theoretical study, we assume a contention-free MAC protocol, where interference from simultaneous transmission can be effectively minimized or avoided. Many sensor network applications (particularly those for long-term monitoring) are likely to operate at low rates. For such low bit rate traffic, a contention-free MAC protocol is fairly easy to design (see, e.g., [16]) and its discussion is beyond the scope of this paper.

Each node $i \in \mathcal{N}$ is initially provisioned with an amount of energy e_i. The base station is not constrained with energy. In this study, network lifetime is defined as the first time instance when any of the sensor nodes runs out of energy. From (1), (2), and (3), it is easy to understand that the location of the base station and the corresponding multi-hop flow routing among the nodes will determine energy consumption behavior at each node and thus the network lifetime. Table I lists all notation used in this paper.

B. Problem Description

The focus of this paper is to investigate how to optimally move a mobile base station to collect real time data in a sensor network so that the network lifetime can be maximized. Note that the network lifetime problem has attracted great interest even for static (fixed) base station problem (see, e.g., [2], [3], [13])

As a first step, we consider the case when the base station is only allowed to be present at a set of pre-determined M positions, denoted as $p_1, p_2, \cdots, p_M$. We call this problem as constrained mobile base station (C-MB) problem. Results on C-MB problem will help us devise solution to the general problem where the base station is allowed to roam anywhere on the two-dimensional plane. We term the latter problem unconstrained mobile base station (U-MB) problem.

Denote $(x, y)(t)$ the position of base station B at time t and T the network lifetime (which is the objective of our optimization problem). Then a feasible flow routing solution realizing this network lifetime T must satisfy both flow conservation and energy constraint at each sensor node. These constraints can be formally stated as follows. Denote $g_{ij}(t)$ and $g_{iB}(t)$ the data rates from node i to node j and base station B at time t, respectively. Under multi-hop multi-path routing, the flow conservation for each node $i \in \mathcal{N}$ at any time $t \in [0,T]$ is

$$\sum_{k\in\mathcal{N}}^{k\neq i} g_{ki}(t) + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} g_{ij}(t) + g_{iB}(t) \ ,$$

i.e., for node i, the sum of total incoming flow rates plus self-generated data rate is equal to the sum of total outgoing flow rates at time t. The energy constraint for each node $i \in \mathcal{N}$ is

$$\int_0^T \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot g_{ki}(t) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot g_{ij}(t) + c_{iB}(t) \cdot g_{iB}(t) \right] dt \leq e_i \ ,$$

i.e., total consumed energy due to reception and transmission over time T cannot exceed its initial energy e_i. By (2), we have

$$c_{iB}(t) = \alpha + \beta \left[\sqrt{(x(t)-x_i)^2 + (y(t)-y_i)^2} \right]^n \ ,$$

where (x_i, y_i) is the location of node i.

Denote $\mathcal{A}$ the search space for the base station, which can be narrowed down to the smallest enclosing disk (SED) for all nodes in the network (see Lemma 1 in Section V). The general U-MB problem can be formulated as follows.

$$\begin{array}{lll}
\textbf{Max} & T & \\
\textbf{s.t.} & \sum_{k\in\mathcal{N}}^{k\neq i} g_{ki}(t) + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} g_{ij}(t) + g_{iB}(t) & (i\in\mathcal{N}, 0\leq t\leq T) \\
& \int_0^T \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot g_{ki}(t) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot g_{ij}(t) + c_{iB}(t) \cdot g_{iB}(t) \right] dt \leq e_i & (i \in \mathcal{N}) \\
& c_{iB}(t) = \alpha + \beta \left[\sqrt{(x(t)-x_i)^2 + (y(t)-y_i)^2} \right]^n & (i\in\mathcal{N}, 0\leq t\leq T) \\
& (x,y)(t) \in \mathcal{A} & (0 \leq t \leq T) \\
& T, g_{ij}(t), g_{iB}(t) \geq 0 & (i,j \in \mathcal{N}, i \neq j, 0 \leq t \leq T)
\end{array}$$

In the above formulation, the base station location (i.e., $(x,y)(t)$ for $0 \leq t \leq T$) and the corresponding flow routing (i.e., $g_{ij}(t)$ and $g_{iB}(t)$ for $0 \leq t \leq T$) form a joint optimization space for the objective T. This formulation is in the form of *non-polynomial programming*. Since even a simpler non-linear programming problem is NP-hard [6], we conclude that the above formulation is NP-hard.

III. From Time Domain to Space Domain

The difficulty of the formulated problem in last section resides in that base station location $(x,y)(t)$ and flow routing $g_{ij}(t)$ and $g_{iB}(t)$ are all functions of time. In this section, we show that as far as network lifetime performance is concerned, such dependency on time can be relaxed. Specifically, we will show (Theorem 1) that the flow routing is only dependent on the location of the base station and is independent of when the base station is present at this location. Further, as long as the total time span for the base station to be present at this location is the same, the specific time instance (i.e., "when") the base station visits this location is not important.

To begin, we define an indicator function $1^+\{\text{event}\}$ as 1 if event is true and 0 otherwise. Denote $W(p)$ the cumulative time span for the base station B to be present at location p under a solution φ, i.e.,

$$W(p) = \int_0^T 1^+\{(x,y)(t) = p\} dt \ .$$

We have the following theorem.

Theorem 1: Denote T^* the maximum network lifetime achieved by an optimal solution φ^* with a base station moving path $(x,y)^*(t)$ and a flow routing $g^*_{ij}(t)$ and $g^*_{iB}(t)$. There exists an equivalent solution $\bar{\varphi}^*$ with the same $W^*(p)$ and time-independent flow rates

$$f^*_{ij}(p) = \frac{\int_0^T g_{ij}(t) \cdot 1^+\{(x,y)^*(t) = p\} dt}{W^*(p)}$$

$$f^*_{iB}(p) = \frac{\int_0^T g_{iB}(t) \cdot 1^+\{(x,y)^*(t) = p\} dt}{W^*(p)}$$

that yields the same maximum network lifetime. Under $f^*_{ij}(p)$ and $f^*_{iB}(p)$, as long as $W^*(p)$ remain the same, the network lifetime T^* will remain unchanged regardless of the ordering and specific time instances when the base station visits each point p.

Note that we have transformed the time dependent solution φ^* (i.e., $(x,y)^*(t)$, $g^*_{ij}(t)$, and $g^*_{iB}(t)$) into a location dependent solution $\bar{\varphi}^*$ (i.e., $W^*(p)$, $f^*_{ij}(p)$, and $f^*_{iB}(p)$). Solution $\bar{\varphi}^*$ can be regarded as the average of solution φ^* at each point p. Based on this insight and the flow conservation of solution φ^*, we can verify the flow conservation for solution $\bar{\varphi}^*$ at each point p. Moreover, we can verify that at time T^*, the energy consumption behavior in solution $\bar{\varphi}^*$ is the same as that in solution $\bar{\varphi}^*$, i.e., solution $\bar{\varphi}^*$ has the same maximum network lifetime. The details of proof can be found in [14].

Based on Theorem 1, we conclude that as far as network lifetime is concerned, it is sufficient for us to obtain location-dependent solution $\bar{\varphi}^*$, which includes $W(p)$, $f_{ij}(p)$, and $f_{iB}(p)$ values when the base station is at each point p. The specific time-dependent realization for $(x,y)(t)$ is not important and such realizations are certainly not unique. As a result, we can transform the optimization problem from time-dependent functions $(x(t,y)(t), g_{ij}(t), g_{iB}(t))$ to location (space)-dependent functions $(W(p), f_{ij}(p), f_{iB}(p))$. Subsequently, U-MB problem formulation given in Section II-B can be reformulated as follows.

$$\begin{array}{ll}
\textbf{Max} & T \\
\textbf{s.t.} & \sum_{p\in\mathcal{A}} W(p) = T \\
\sum_{k\in\mathcal{N}}^{k\neq i} f_{ki}(p) + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij}(p) + f_{iB}(p) & (i \in \mathcal{N}, p\in\mathcal{A}) \\
\sum_{p\in\mathcal{A}} \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot f_{ki}(p) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot f_{ij}(p) + c_{iB}(p) \cdot f_{iB}(p) \right] W(p) \leq e_i & (i \in \mathcal{N}) \qquad (4)
\end{array}$$

$$T, W(p), f_{ij}(p), f_{iB}(p) \geq 0 \quad (i, j \in \mathcal{N}, i \neq j, p \in \mathcal{A})$$

Note that integration $\int_0^T (\cdot)dt$ (with respect to time) in the original problem formulation (Section II-B) has been changed to $\sum_{p\in\mathcal{A}}(\cdot)$ (with respect to space) in the new formulation. This novel transformation will enable us to develop provably approximation algorithm in the space domain, which we will elaborate in Section V.

IV. Optimal Solution to the C-MB Problem

In this section, we show that C-MB problem can be formulated as an LP problem, which can be solved in polynomial time. This result will be useful when we solve the U-MB problem in Section V.

Recall that in the C-MB problem, the location of base station is limited to a given set of pre-determined locations p_m, $m = 1, 2, \cdots, M$. Based on the results in Section III, we need to find optimal time span $W(p_m)$ and the corresponding flow routing $f_{ij}(p_m)$ and $f_{iB}(p_m)$ when the base station is at each p_m to maximize the network lifetime.

When the base station is at point p_m, $1 \leq m \leq M$, the flow conservation for node $i \in \mathcal{N}$ is

$$\sum_{k\in\mathcal{N}}^{k\neq i} f_{ki}(p_m) + r_i = \sum_{j\in\mathcal{N}}^{j\neq i} f_{ij}(p_m) + f_{iB}(p_m) \ . \tag{5}$$

The energy constraint for node $i \in \mathcal{N}$ is

$$\sum_{m=1}^{M} \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot f_{ki}(p_m) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot f_{ij}(p_m) + c_{iB}(p_m) \cdot f_{iB}(p_m) \right] W(p_m) \leq e_i \ . \tag{6}$$

Note that for each i and p_m, $c_{iB}(p_m)$ is a constant.

We can formulate C-MB problem as an LP by letting $V_{ij}(p_m) = f_{ij}(p_m) \cdot W(p_m)$ and $V_{iB}(p_m) = f_{iB}(p_m) \cdot W(p_m)$, where $V_{ij}(p_m)$ (or $V_{iB}(p_m)$) can be interpreted as the total data volume from sensor node i to sensor node j (or base station B) when the base station is at p_m. We have

LP(C-MB) **Max** T

s.t. $$\sum_{m=1}^{M} W(p_m) - T = 0$$

$$\sum_{k\in\mathcal{N}}^{k\neq i} V_{ki}(p_m) + r_i \cdot W(p_m) - \sum_{j\in\mathcal{N}}^{j\neq i} V_{ij}(p_m) - V_{iB}(p_m) = 0 \quad (i \in \mathcal{N}, 1 \leq m \leq M) \tag{7}$$

$$\sum_{m=1}^{M} \left[\sum_{k\in\mathcal{N}}^{k\neq i} \rho \cdot V_{ki}(p_m) + \sum_{j\in\mathcal{N}}^{j\neq i} C_{ij} \cdot V_{ij}(p_m) + c_{iB}(p_m) \cdot V_{iB}(p_m) \right] \leq e_i \quad (i \in \mathcal{N}) \tag{8}$$

$$T, W(p_m), V_{ij}(p_m), V_{iB}(p_m) \geq 0 \quad (i, j \in \mathcal{N}, i \neq j, 1 \leq m \leq M) \ ,$$

where (7) and (8) follow from (5) and (6), respectively. Once we solve the above LP, we can obtain $f_{ij}(p_m)$ and $f_{iB}(p_m)$ by $f_{ij}(p_m) = \frac{V_{ij}(p_m)}{W(p_m)}$ and $f_{iB}(p_m) = \frac{V_{iB}(p_m)}{W(p_m)}$. We summarize the result in this section with the following proposition.

Proposition 1: C-MB problem can be solved via a single LP in polynomial time.

V. A $(1-\varepsilon)$ Optimal Algorithm to the U-MB Problem

Building upon the results in the previous sections, we are now ready to present the main result of this paper.

A. Subareas and Fictitious Cost Points

Motivation. Under the U-MB problem, the base station can move to any point in the two-dimensional plane. Clearly, in order to maximize network lifetime, we would expect that the location for the base station can be narrowed down to the *smallest enclosing disk* (SED) [19], which is a disk with the smallest radius that contains all the sensor nodes in the network. This is formally stated in the following lemma.

Lemma 1: To maximize the network lifetime for the U-MB problem, the base station must stay within the smallest enclosing disk $\mathcal{A}$ that covers all the sensor nodes in the network.

The proof is based on contradiction and we omit it here to conserve space. It has been shown in [9] that SED is unique and can be found in $O(N)$ time, where N is the number of nodes to be covered.

Although we have narrowed down the search space for base station B from an infinite two-dimensional plane to SED $\mathcal{A}$, there are still infinite number of points in $\mathcal{A}$. To obtain a finite search space, we consider dividing $\mathcal{A}$ into small subareas, $\mathcal{A}_1, \mathcal{A}_2, \cdots$, up to say $\mathcal{A}_M$, i.e.,

$$\mathcal{A} = \bigcup_{m=1}^{M} \mathcal{A}_m \ .$$

For approximation, it is tempting to use a point $q_m \in \mathcal{A}_m$, $m = 1, 2, \cdots, M$, to represent subarea $\mathcal{A}_m$. Indeed, when each subarea is sufficiently small, we may use some point $q_m \in \mathcal{A}_m$ to represent $\mathcal{A}_m$, $m = 1, 2, \cdots, M$, and apply the LP approach in Section IV on these M points to get a very good solution.

However, such approach is *heuristic* at best and unfortunately does not provide any *theoretical guarantee* on performance. A theoretical question is how to divide M subareas on $\mathcal{A}$ such that an algorithm can be developed that yields *provably* $(1-\varepsilon)$ optimal network lifetime performance, i.e., a network lifetime that is guaranteed to be at least $(1-\varepsilon)$ of the optimum.

Our Approach. Our approach is to examine the energy constraint in (4) and exploit how the location of the base station affect the energy consumption. Note that the location of the base station is *embedded* in the cost parameter c_{iB}. Thus, to design a $(1-\varepsilon)$ optimal algorithm, we may consider dividing disk $\mathcal{A}$ into subareas, with each subarea to be associated with some nice properties on c_{iB}'s that can be used to prove $(1-\varepsilon)$ optimality.

Denote $O_{\mathcal{A}}$ and $R_{\mathcal{A}}$ the origin and radius of the SED $\mathcal{A}$. For each sensor node $i \in \mathcal{N}$, denote $D_{i,O_{\mathcal{A}}}$ the distance from sensor node i to the origin of disk $\mathcal{A}$. Denote $D_{iB}^{\min}$ and $D_{iB}^{\max}$ the minimum and maximum distance between sensor node i and base station B, respectively; denote $C_{iB}^{\min}$ and $C_{iB}^{\max}$ the corresponding minimum and maximum cost between sensor node i and base station B, respectively. Then, since the search space for base station B is now narrowed down to disk $\mathcal{A}$ (see Fig. 1), we have

$$\begin{aligned} D_{iB}^{\min} &= 0 , \\ D_{iB}^{\max} &= D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}} . \end{aligned}$$

By (2), we have

$$\begin{aligned} C_{iB}^{\min} &= \alpha , && (9) \\ C_{iB}^{\max} &= \alpha + \beta \cdot (D_{iB}^{\max})^n = \alpha + \beta \cdot (D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}})^n . && (10) \end{aligned}$$

Given the range of $d_{iB} \in [D_{iB}^{\min}, D_{iB}^{\max}]$, for each sensor node $i \in \mathcal{N}$, we now show how to divide disk $\mathcal{A}$ into a set of *non-uniform* subareas with the distance of each subarea to sensor node i meeting some properties that can be used to design $(1-\varepsilon)$ optimal algorithm.

Specifically, for each sensor node $i \in \mathcal{N}$, we draw a sequence of circles centered at sensor node i, each with increasing radius $D[1], D[2], \cdots, D[H_i]$ corresponding to costs $C[1], C[2], \cdots, C[H_i]$ that are defined as follows.

$$C[h] = C_{iB}^{\min}(1+\varepsilon)^h = \alpha(1+\varepsilon)^h \quad (1 \le h \le H_i) \qquad (11)$$

The number of required circles H_i can be determined by having the last circle in the sequence (with radius $D[H_i]$) to completely include disk $\mathcal{A}$, i.e. $D[H_i] > D_{iB}^{\max}$, or equivalently,

$$C[H_i] \ge C_{iB}^{\max} .$$

We have

$$\begin{aligned} H_i &= \left\lceil \frac{\ln(C_{iB}^{\max}/C_{iB}^{\min})}{\ln(1+\varepsilon)} \right\rceil \\ &= \left\lceil \frac{\ln(1+\frac{\beta}{\alpha}(D_{i,O_{\mathcal{A}}}+R_{\mathcal{A}})^n)}{\ln(1+\varepsilon)} \right\rceil = O\left(\frac{1}{\varepsilon}\right) . \qquad (12) \end{aligned}$$

That is, we have a total of H_i circles with center at sensor node i, each with cost $C[h]$, $h = 1, 2, \cdots, H_i$. These H_i circles provide H_i non-overlapping rings. Now suppose base station B is moved to any point in the h-th ring, $h = 1, 2, \cdots, H_i$. Then we have

$$C[h-1] \le c_{iB} \le C[h] , \qquad (13)$$

where we define $C[0] = C_{iB}^{\min} = \alpha$.

We perform the above process for each sensor node $i \in \mathcal{N}$. The intersecting circles will divide disk $\mathcal{A}$ into a number of non-uniform subareas, with the boundaries of each subarea being either an arc centered at some sensor node $i \in \mathcal{N}$ (with some cost $C[h]$, $1 \le h < H_i$) or an arc of disk $\mathcal{A}$. As an example, disk $\mathcal{A}$ in Fig. 2 is now divided into 27 subareas.

So what nice properties do we have about dividing $\mathcal{A}$ into these non-uniform subareas? We will show that for a point in each of these subareas, its cost to each sensor node can be *tightly* bounded from both above and below. As a result, this can be exploited to design a $(1-\varepsilon)$ optimal algorithm. Note that for each sensor node $i \in \mathcal{N}$, any subarea $\mathcal{A}_m$ must be within a ring with center at sensor node i. Denote the index of this ring as $h_i(\mathcal{A}_m)$. That is, when the base station B is at any point $p \in \mathcal{A}_m$, we have

$$C[h_i(\mathcal{A}_m) - 1] \le c_{iB}(p) \le C[h_i(\mathcal{A}_m)] \qquad (14)$$

by (13). Since $\frac{C[h_i(\mathcal{A}_m)]}{C[h_i(\mathcal{A}_m)-1]} = 1+\varepsilon$ by (11), these two bounds for $c_{iB}(p)$ are very tight.

To use the result in Section IV, we introduce the notion of *fictitious cost point* p_m to represent each subarea $\mathcal{A}_m$, $m = 1, 2, \cdots, M$. The fictitious cost point is an N-tuple vector embodying the upper cost bound for any point within this subarea $\mathcal{A}_m$ to all the N sensor nodes in the network. Specifically, denote the N-tuple cost vector for fictitious cost point p_m (corresponding to subarea $\mathcal{A}_m$) as $[c_{1B}(p_m), c_{2B}(p_m), \cdots, c_{NB}(p_m)]$, with the i-th component $c_{iB}(p_m)$ being

$$c_{iB}(p_m) = C[h_i(\mathcal{A}_m)] , \qquad (15)$$

where $h_i(\mathcal{A}_m)$ is determined by (14).

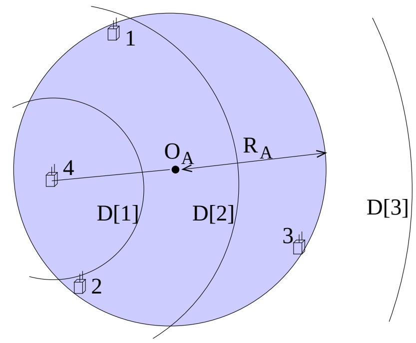

Fig. 1. A sequence of circles with increasing costs with center at node 4.

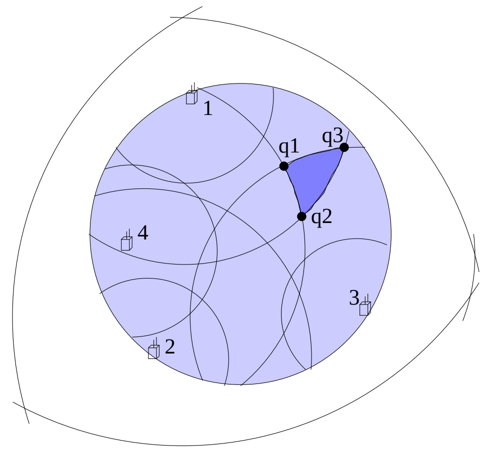

Fig. 2. An example of subareas within disk $\mathcal{A}$ that are obtained by intersecting arcs from different circles.

As an example, the fictitious cost point for subarea with corner points (q_1, q_2, q_3) in Fig. 2 can be represented by 4-tuple cost vector $[c_{1B}(p_m), c_{2B}(p_m), c_{3B}(p_m), c_{4B}(p_m)] = [C[2], C[3], C[2], C[3]]$, where the first component $C[2]$ represents an upper bound of cost for any point in this subarea to sensor node 1, the second component $C[3]$ represents an upper bound of cost (which is loose here) for any point in this subarea to sensor node 2, and so forth.

We have the following property for fictitious cost points.

Property 1: Denote p_m the fictitious cost point for subarea $\mathcal{A}_m$, $m = 1, 2, \cdots, M$. For any point $p \in \mathcal{A}_m$, we have $c_{iB}(p) \le c_{iB}(p_m) \le (1+\varepsilon) \cdot c_{iB}(p)$.

Proof. By (14) and the definition of fictitious cost point p_m (see (15)), we have $c_{iB}(p) \le c_{iB}(p_m)$. Further, we have

$$\begin{aligned} c_{iB}(p_m) &= C[h_i(\mathcal{A}_m)] \\ &= (1+\varepsilon) \cdot C[h_i(\mathcal{A}_m)-1] \le (1+\varepsilon) \cdot c_{iB}(p) , \end{aligned}$$

where the last inequality follows from (14). This completes the proof. □

Now the set of M non-uniform subareas are represented by the M fictitious cost points, with each fictitious cost point having an N-tuple cost vector to all the N sensor nodes in the network. Although a fictitious cost point may not be mapped to a physical point, using these fictitious cost points will aid the design of a $(1-\varepsilon)$ optimal algorithm. Note that for network lifetime problems, we only need to consider the cost terms c_{iB} for $i = 1, 2, \cdots, N$, which is exactly captured by the N-tuple representation for each fictitious cost point. As a result, we can now readily apply the LP approach discussed in Section IV to formulate an optimization problem on these M fictitious cost points. In the next section, we will show that an optimal solution to the C-MB problem on fictitious cost points can be used to construct a $(1-\varepsilon)$ optimal solution to the U-MB problem.

B. $(1-\varepsilon)$ Optimality

Denote $\psi^*_{\text{U-MB}}$ an optimal solution to the U-MB problem and $T^*_{\text{U-MB}}$ the corresponding maximum network lifetime, both of which are unknown. Our objective is to find a solution to the U-MB problem that has provably $(1-\varepsilon)$ optimal network lifetime. Denote $\psi^*_{\text{C-MB}}$ an optimal solution to the C-MB problem obtained by applying an LP on the M fictitious cost points p_m, $m = 1, 2, \cdots, M$, and $T^*_{\text{C-MB}}$ the corresponding network lifetime.

Our roadmap for theoretical proofs is as follows. In Theorem 2, we will prove that $T^*_{\text{C-MB}} \ge (1-\varepsilon) T^*_{\text{U-MB}}$ (see Fig. 3). Since the optimal solution $\psi^*_{\text{C-MB}}$ corresponding to $T^*_{\text{C-MB}}$ are based on the M fictitious cost points instead of physical points, in Theorem 3 we will further show how to construct a solution $\psi_{\text{U-MB}}$ to the U-MB problem based on $\psi^*_{\text{C-MB}}$ and prove that the corresponding network lifetime is $(1-\varepsilon)$ optimal, i.e., $T_{\text{U-MB}} \ge (1-\varepsilon) T^*_{\text{U-MB}}$ (see Fig. 3).

Theorem 2: For a given $\varepsilon > 0$, define subareas $\mathcal{A}_m$ and fictitious cost points p_m, $m = 1, 2, \cdots, M$, as in Section V-A. Then we have $T^*_{\text{C-MB}} \ge (1-\varepsilon) \cdot T^*_{\text{U-MB}}$.

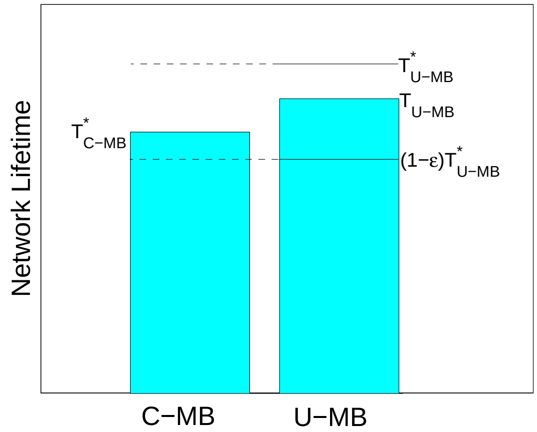

Fig. 3. Comparison of network lifetimes under different solutions used to construct $(1-\varepsilon)$ optimal solution.

To prove that Theorem 2 is true, we need the following lemma, where subareas $\mathcal{A}_m$ and fictitious cost points p_m, $m = 1, 2, \cdots, M$ are defined the same as in Theorem 2 for a given $\varepsilon > 0$. Denote $W(\mathcal{A}_m)$ the cumulative time period when the base station B is present within subarea $\mathcal{A}_m$ for the U-MB problem, i.e., $W(\mathcal{A}_m) = \sum_{p \in \mathcal{A}_m} W(p)$.

Lemma 2: Suppose we have a given solution $\pi_{\text{U-MB}}$ to the U-MB problem with $W(p)$, $f_{ij}(p)$, $f_{iB}(p)$, and a network lifetime $T_{\text{U-MB}}$. For a given $\varepsilon > 0$, we can always construct a solution $\pi_{\text{C-MB}}$ to the C-MB problem on these fictitious cost points such that network lifetime $T_{\text{C-MB}} \ge (1-\varepsilon) \cdot T_{\text{U-MB}}$ by having the base station stay $W(p_m)$ amount of time on fictitious cost point p_m, where

$$W(p_m) = (1-\varepsilon) \cdot W(\mathcal{A}_m) \tag{16}$$

and setting the flow routing on p_m as

$$f_{ij}(p_m) = \frac{\sum_{p \in \mathcal{A}_m} f_{ij}(p) W(p)}{W(\mathcal{A}_m)} , \tag{17}$$

$$f_{iB}(p_m) = \frac{\sum_{p \in \mathcal{A}_m} f_{iB}(p) W(p)}{W(\mathcal{A}_m)} . \tag{18}$$

Lemma 2 can be proved by showing that flow conservation holds in solution $\pi_{\text{C-MB}}$ and at time $(1-\varepsilon) \cdot T_{\text{U-MB}}$, the energy constraint holds at each node. The details of proof can be found in [14].

Lemma 2 is a powerful result. It states that for any given solution $\pi_{\text{U-MB}}$ to the U-MB problem, we can find a solution $\pi_{\text{C-MB}}$ for the set of fictitious cost points (corresponding to a given ε), such that the network lifetime $T_{\text{C-MB}}$ is at least $(1-\varepsilon)$ of $T_{\text{U-MB}}$. Now we are ready to prove Theorem 2.

Proof of Theorem 2. Consider the special case of Lemma 2 that the given solution to U-MB problem, $\pi_{\text{U-MB}}$, is an optimal solution with network lifetime $T^*_{\text{U-MB}}$. We can transform it into a solution to the C-MB problem on fictitious cost points with network lifetime at least $(1-\varepsilon) T^*_{\text{U-MB}}$, i.e., there is a solution to C-MB problem on fictitious cost points with network lifetime of at least $(1-\varepsilon) T^*_{\text{U-MB}}$. As a result, the solution $\psi^*_{\text{C-MB}}$ to

C-MB problem on fictitious cost points must have a network lifetime $T^*_{\text{C-MB}} \geq (1-\varepsilon)T^*_{\text{U-MB}}$. □

Theorem 2 guarantees that the network lifetime obtained from the LP solution on the M fictitious cost points is at least $(1-\varepsilon)$ of $T^*_{\text{U-MB}}$. However, a fictitious cost point may not be mapped to a physical point, which is required in the final solution. In the following theorem, we show that if we have an optimal solution to the C-MB problem based on fictitious cost points, we can construct a solution with each point being physically realizable. Further, the network lifetime for this constructed solution is greater than or equal to the maximum network lifetime for the C-MB problem, i.e., $T_{\text{U-MB}} \geq T^*_{\text{C-MB}}$. As a result, this new solution is $(1-\varepsilon)$ optimal.

Theorem 3: For a given $\varepsilon > 0$, define subareas $\mathcal{A}_m$ and fictitious cost points p_m, $m = 1, 2, \cdots, M$, as discussed in Section V-A. Given an optimal solution $\psi^*_{\text{C-MB}}$ on these M fictitious cost points with $W^*(p_m)$, $f^*_{ij}(p_m)$, $f^*_{iB}(p_m)$, and corresponding network lifetime $T^*_{\text{C-MB}}$, a $(1-\varepsilon)$ optimal solution $\psi_{\text{U-MB}}$ to U-MB problem can be constructed by having the base station stay in $\mathcal{A}_m$ for $W(\mathcal{A}_m) = W^*(p_m)$ amount of time with a corresponding flow routing for any point $p \in \mathcal{A}_m$ as $f_{ij}(p) = f^*_{ij}(p_m)$ and $f_{iB}(p) = f^*_{iB}(p_m)$.

Theorem 3 can be proved by first showing that flow conservation holds in $\psi_{\text{U-MB}}$ and the network lifetime of $\psi_{\text{U-MB}}$ is at least $T^*_{\text{C-MB}}$. Then since $T^*_{\text{C-MB}} \geq (1-\varepsilon)T^*_{\text{U-MB}}$ (see Theorem 2), we have that $T_{\text{U-MB}}$ is at least $(1-\varepsilon)T^*_{\text{U-MB}}$, i.e., $\psi_{\text{U-MB}}$ is $(1-\varepsilon)$ optimal. The details of proof can be found in [14].

C. Summary of Algorithm and An Example

The design of the $(1-\varepsilon)$ optimal algorithm is described in Sections V-A and V-B. We now summarize it into the following algorithm.

Algorithm 1: **(A $(1-\varepsilon)$ Optimal Algorithm)**

1) *Within the smallest enclosing disk $\mathcal{A}$, compute the lower and upper cost bounds C^{min}_{iB} and C^{max}_{iB} for each node $i \in \mathcal{N}$ by (9) and (10).*
2) *For a given $\varepsilon > 0$, define a sequence of costs $C[1]$, $C[2], \cdots, C[H_i]$ by (11), where H_i is defined by (12).*
3) *For each node $i \in \mathcal{N}$, draw a sequence of $(H_i - 1)$ circles corresponding to cost $C[h]$ centered at node i, $1 \leq h < H_i$. The intersection of these circles within disk $\mathcal{A}$ will divide $\mathcal{A}$ into M subareas $\mathcal{A}_1, \mathcal{A}_2, \cdots, \mathcal{A}_M$.*
4) *For each subarea $\mathcal{A}_m$, $1 \leq m \leq M$, define a fictitious cost point p_m, which is represented by N-tuple cost vector $[c_{1B}(p_m), c_{2B}(p_m), \cdots, c_{NB}(p_m)]$, where $c_{iB}(p_m)$ is defined in (15).*
5) *For the C-MB problem on these M fictitious cost points, apply the LP formulation in Section IV and obtain an optimal solution $\psi^*_{\text{C-MB}}$ with $W^*(p_m)$, $f^*_{ij}(p_m)$, and $f^*_{iB}(p_m)$.*
6) *Construct a $(1-\varepsilon)$ optimal solution $\psi_{\text{U-MB}}$ to U-MB problem based on $\psi^*_{\text{C-MB}}$ using the procedure in Theorem 3.*

In the above algorithm, Step 5 has the highest complexity (solving an LP). Since there are $(H_i - 1)$ circles radiating from sensor node $i \in \mathcal{N}$ and one circle for disk $\mathcal{A}$, the total number of subareas M through the intersection of these circles is upper bounded by $O([1 + \sum_{i=1}^{N}(H_i - 1)]^2) = O((N/\varepsilon)^2)$. Thus, the LP in Step 5 has polynomial size and the complexity of the above algorithm is polynomial.

TABLE II
SENSOR LOCATIONS, DATA RATE, AND INITIAL ENERGY OF THE EXAMPLE SENSOR NETWORK

Node Index	(x_i, y_i)	r_i	e_i
1	$(0.1, 0.5)$	0.8	390
2	$(1.1, 0.7)$	1.0	400
3	$(0.4, 0.1)$	0.6	130

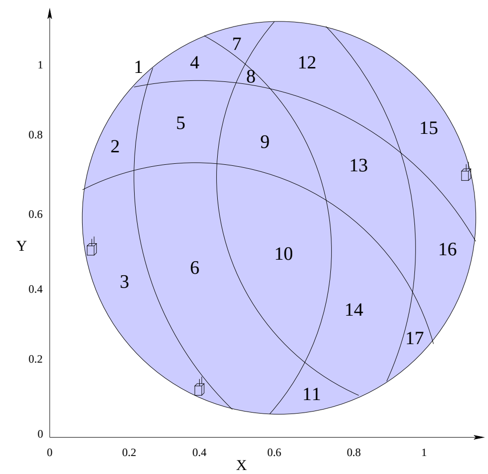

Fig. 4. The subareas for the example sensor network.

Example 1: To illustrate the steps in Algorithm 1, we solve a small 3-sensor network problem as an example. The location, data rate, and initial energy for each sensor are shown in Table II, where the units of distance, rate, and energy are all normalized. We use $n = 2$ in this example and the network setting are $\alpha = 1$, $\beta = 0.5$ and $\rho = 1$ under normalized units. For illustration, we set $\varepsilon = 0.2$.[1]

In Step 1, we first identify SED $\mathcal{A}$ with origin $O_{\mathcal{A}} = (0.61, 0.57)$ and radius $R_{\mathcal{A}} = 0.51$ (see Fig. 4). Then we have $D_{i,O_{\mathcal{A}}} = R_{\mathcal{A}} = 0.51$ for each node i, $1 \leq i \leq 3$. We then find the lower and upper bounds of c_{iB} for each node i as follows.

$$\begin{aligned} C^{\min}_{iB} &= \alpha = 1 \\ C^{\max}_{iB} &= \alpha + \beta(D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}})^n \\ &= 1 + 0.5 \cdot (0.51 + 0.51)^2 = 1.52 \end{aligned}$$

In Step 2, for $\varepsilon = 0.2$, we find

$$\begin{aligned} H_i &= \left\lceil \frac{\ln(1 + \frac{\beta}{\alpha}(D_{i,O_{\mathcal{A}}} + R_{\mathcal{A}})^n)}{\ln(1+\varepsilon)} \right\rceil \\ &= \left\lceil \frac{\ln(1 + \frac{0.5}{1}(0.51 + 0.51)^2)}{\ln(1+0.2)} \right\rceil = 3 \end{aligned}$$

[1]In Section V-D, we use $\varepsilon = 0.05$ for all numerical results.

for each node i, $1 \le i \le 3$, and

$$\begin{aligned} C[1] &= \alpha(1+\varepsilon) = 1 \cdot (1+0.2) = 1.20 \ , \\ C[2] &= \alpha(1+\varepsilon)^2 = 1 \cdot (1+0.2)^2 = 1.44 \ , \\ C[3] &= \alpha(1+\varepsilon)^3 = 1 \cdot (1+0.2)^3 = 1.73 \ . \end{aligned}$$

In Step 3, we draw circles with centered at each node i, $1 \le i \le 3$, and with cost $C[h]$, $1 \le h < H_i = 3$, to divide the SED $\mathcal{A}$ into 17 subareas $\mathcal{A}_1, \mathcal{A}_2, \cdots, \mathcal{A}_{17}$.

In Step 4, we define a fictitious cost point p_m for each subarea $\mathcal{A}_m$, $1 \le m \le 17$. For example, for fictitious cost point p_1, we define the 3-tuple cost vector as $[c_{1B}(p_m), c_{2B}(p_m), c_{3B}(p_m)] = [C[1], C[3], C[3]] = [1.20, 1.73, 1.73]$.

In Step 5, we obtain an optimal solution $\psi^*_{\text{C-MB}}$ to C-MB problem on these 17 fictitious cost points by the LP approach discussed in Section IV. We obtain the network lifetime $T^*_{\text{C-MB}} = 190.37$, $W^*(p_3) = 157.00$, $W^*(p_6) = 33.37$, and for all other 15 fictitious cost points, we have $W^*(p_m) = 0$ (i.e., base station will not visit those areas). When the base station is at fictitious cost point p_3, the routing is $f^*_{1B}(p_3) = 1.4$, $f^*_{2B}(p_3) = 1.0$, and $f^*_{31}(p_3) = 0.6$. When the base station is at fictitious cost point p_6, the routing is $f^*_{1B}(p_6) = 0.8$, $f^*_{2B}(p_6) = 1.0$, and $f^*_{3B}(p_6) = 0.6$.

In Step 6, we obtain a $(1-\varepsilon)$ optimal solution $\psi_{\text{U-MB}}$ to U-MB problem as follows. Let the base station stay at any point in subarea $\mathcal{A}_3$ for 157.00 time and stay at any point in subarea $\mathcal{A}_6$ for 33.37 time. When the base station is at a point p in subarea $\mathcal{A}_3$, the routing is $f_{1B}(p) = 1.4$, $f_{2B}(p) = 1.0$, and $f_{31}(p) = 0.6$. When the base station is at a point p in subarea $\mathcal{A}_6$, the routing is $f_{1B}(p) = 0.8$, $f_{2B}(p) = 1.0$, and $f_{3B}(p) = 0.6$. The network lifetime for $\psi_{\text{U-MB}}$ is greater than or equal to 190.37 and is $(1-\varepsilon)$ optimal. □

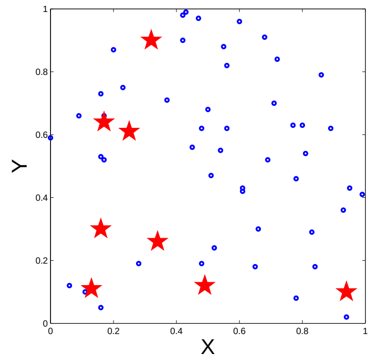

Fig. 5. A 50-node network used in numerical investigation.

$\mathcal{A}_m(x,y)$	$W(\mathcal{A}_m)$
(0.13, 0.11)	0.39
(0.17, 0.64)	1.47
(0.32, 0.90)	26.23
(0.25, 0.61)	27.72
(0.49, 0.12)	3.43
(0.94, 0.10)	9.66
(0.34, 0.26)	8.37
(0.16, 0.30)	45.03

TABLE III
$(1-\varepsilon)$ OPTIMAL RESULTS FOR THE 50-NODE NETWORK WITH $\varepsilon = 0.05$.

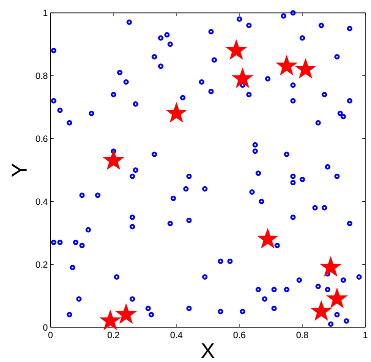

Fig. 6. A 100-node network used in numerical investigation.

$\mathcal{A}_m(x,y)$	$W(\mathcal{A}_m)$
(0.75, 0.83)	0.15
(0.86, 0.05)	0.38
(0.69, 0.28)	50.14
(0.24, 0.04)	2.10
(0.59, 0.88)	0.57
(0.81, 0.82)	0.09
(0.20, 0.53)	21.21
(0.91, 0.09)	0.05
(0.40, 0.68)	41.64
(0.89, 0.19)	3.16
(0.61, 0.79)	23.82
(0.19, 0.02)	6.14

TABLE IV
$(1-\varepsilon)$ OPTIMAL RESULTS FOR THE 100-NODE NETWORK WITH $\varepsilon = 0.05$.

D. Numerical Results

Now we apply the $(1-\varepsilon)$ optimal algorithm for larger sized networks and use numerical results to demonstrate the efficacy of the algorithm. We consider two randomly generated networks consisting of 50 and 100 nodes deployed over a 1×1 square, respectively. The data rate and initial energy for each node are randomly generated between $[0.1, 1]$ and $[50, 500]$, respectively. The units of distance, rate, and energy are all normalized appropriately. The normalized parameters in energy consumption model are $\alpha = \beta = \rho = 1$. We assume the path loss index $n = 2$.

The required accuracy for approximation algorithm ε is set to $\varepsilon = 0.05$ for all numerical results. That is, we are pursuing a solution with a network lifetime that is at least 95% of the maximum network lifetime.

The network topology for the 50-node network is shown in Fig. 5, where a circle represents a sensor node and a star represents an optimal location for base station. We omit to list the coordinates of each node due to paper length limitation. By applying Algorithm 1, we obtain a $(1-\varepsilon)$ optimal network lifetime 122.30, which is guaranteed to be at least 95% of the optimum. In Table III, we have 8 locations that will be visited by the base station in the $(1-\varepsilon)$ optimal solution and the corresponding time span for the base station to stay in each of these 8 subareas. For illustration purpose, we use a point (x, y) within a subarea $\mathcal{A}_m$ to represent the approximate location of this subarea. For example, we use the point $(0.13, 0.11)$ to represent the subarea that contains this point.

It is worth noting that for 95% of accuracy in optimality, there are only 8 subareas for the base station to visit. It turns out that even for the 100-node network, the number of subareas that needs to be visited by the base station is still very small (12 subareas). This new observation is not obvious. But it is a good news as it hints that the base station does not need to be involved in frequent movement to achieve near-optimal solution.

For the 100-node network, the positions of the nodes are shown in Fig. 6. By applying Algorithm 1, we obtain a $(1-\varepsilon)$ optimal network lifetime 149.45. Again, we use a point (x, y) within a subarea $\mathcal{A}_m$ to represent the approximate location of this subarea. For this particular 100-node network setting, we have 12 subareas that the base station should visit in the final solution, with the corresponding time span in each subarea listed in Table IV.

VI. Related Work

Energy efficient routing has been an active area of research for sensor network in recent years (see, e.g., [12], [15], [17], [18]). It is now well understood that energy efficient routing differs from lifetime-optimal routing as the former advocates the use of minimum energy-cost path, which may overload nodes along some common shared path, leading to poor performance in network lifetime.

Routing algorithms to maximize network lifetime has been an active area of research even for fixed base station location (see, e.g., [2], [3], [13] and references therein). The focus is mainly devoted to how to split traffic flow along different routes and how to adjust power level at each node so that some optimal flow routing topology can be set up to maximize network lifetime. These early works have laid foundation on the importance of power control and flow routing topology on network lifetime performance.

There are some recent work on optimal base station placement [4], [10]. The focus of these efforts is to find an optimal *fixed* position for the base station so that network lifetime can be maximized. However, as pointed out in [8], [20], network lifetime can be substantially increased if the optimization problem can be expanded to include movement of the base station during the course of sensor network operation.

Relevant work in the area of mobile base station for network lifetime problems include [1], [5], [8], [20]. In [1], [5], the locations of base station are constrained on a set of "pre-determined" locations. In [20], Younis et al. showed that mobile base station can increase network lifetime. In [8], Luo and Hubaux proposed to minimize the maximum load on a node among all the nodes in the network, which can be considered as an equivalent problem to maximize network lifetime. The results in [8], [20] are *heuristic*, and thus do not provide any theoretical bound on network lifetime performance.

VII. Conclusion

The benefits of employing mobile base station to prolong sensor network lifetime are significant. However, due to the complexity of the problem, theoretical results have remained an open problem. This paper fills this theoretical gap by contributing a provably optimal algorithm regarding mobile base station. We first showed a novel time-to-space transformation for problem formulation. Based on this transformation, we showed that when the location of the base station are constrained to be on a set of pre-determined points, the optimal solution can be obtained via a single LP. Building upon these results, we further showed that for the general mobile base station problem where the location of the base station is unconstrained, we can design an approximation algorithm such that the network lifetime is guaranteed to be at least $(1-\varepsilon)$ of the maximum network lifetime, where $\varepsilon > 0$ can be made arbitrarily small depending on required precision.

Acknowledgment

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